MSFC FORM 422 (VERTICAL) (AUGUST 1960)

D004

APPLICATION		PART No.	MF	REVISIONS			
NEXT ASSY	USED ON			SYM	DESCRIPTION	DATE	APPROVAL
				A	REVISED	5/14/73	
		11/2/79		B	"	9/25/79	
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		1/15/92		E	Incorporate EO #2	12/19/91	

ASSESSMENT OF FLEXIBLE LINES

FOR

FLOW INDUCED VIBRATION

5-14-73

UNLESS OTHERWISE SPECIFIED DIMENSIONS ARE IN INCHES TOLERANCES ON: FRACTIONS DECIMALS ANGLES	ORIGINAL DATE OF DRAWING 4-17-73; DRAFTSMAN; CHECKER HANDGREN; TRACER; CHECKER R. BURFORD; ENGINEER; ENGINEER	ASSESSMENT OF FLEXIBLE LINES FOR FLOW INDUCED VIBRATION	GEORGE C. MARSHALL SPACE FLIGHT CENTER NATIONAL AERONAUTICS AND SPACE ADMINISTRATION HUNTSVILLE, ALABAMA
MATERIAL 5/9/73	SUBMITTED 11 MAY 73	SCALE	DWG SIZE A — 20M02540
HEAT TREATMENT	APPROVED 5/11/73		SHEET 1 OF 107
FINAL PROTECTIVE FINISH			

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REVISION LOG

Revision Letter	Date of Revision	Revised Pages	Description
Baseline	4/17/73		
A	5/14/73		
B	9/25/79		
C	8/18/87	ALL	Completely revised to reflect new analytical procedures developed in NASA TM-82556.
D (3/1/90 RELEASE)	2/28/90	ALL	Completely revised. Major changes include convolute bending mode, new examples, new computer program (ver. 3.2), new FNCO eqn., added safety factors, added oper. velocity criteria, corrected static stress eqns., added modified Goodman method, and general clarification.
E (1/15/92 RELEASE)	12/19/91	3-11,14, 15,26,29, 33,34,36, 37,39,44, 40,41,45, 46,48-58, 61,63-69, 72,74,75, 77-86,90, 93-100, 103-106, 108, 110-114, 116-118	Modified scope of the document. Changed "safety factor" to "uncertainty factor." Deleted static stress eqns. (App. B). Added a system level analysis (para. 3.0). Deleted materials data (Tables 2 & 3). Changed method of fatigue assessment. Deleted modified Goodman approach. Added two references. Made minor changes to computer program. Made several other minor changes. NOTE: Refer to EO #2 for detailed description of changes.

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1.0 GENERAL

It is well known that the occurrence of flow-induced vibrations in flexible lines, specifically metal bellows and flexhoses, can cause premature failure. This is attributed to a resonance caused by the coupling of vortex shedding from the convolutes with the natural frequencies of the flexible line. A goal in designing these bellows and flexhoses is to prevent resonance from occurring. In the event this goal cannot be met, it is then desirable to analytically predict what the expected life of the bellows and flexhose is due to flow-induced vibration loads.

1.1 Scope

The purpose of this document is to establish the analytical methods for determining whether a given design of an annular convoluted metal bellows or flexhose is susceptible to flow-induced vibrations. These analytical methods include predicting the excitation flow range, frequency, and the corresponding stress resulting from only flow-induced vibration loads. This then leads to prediction of the expected life of the bellows or flexhose, with a final objective of achieving a theoretically infinite life for flow-induced vibrations.

The analytical assessment in this document shall be performed on all flexible lines consisting of formed annular

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convoluted metal bellows or flexhose, except those contained in paragraph 1.2, regardless of fluid velocity. It does not consider other bellows or flexhose configurations such as welded disc, ring reinforced, toroidal, etc. For those type configurations which do not fit this analysis, some other approved analysis or testing must be done.

The analytical model does not account for changes in the flexible line during thermal transients. Therefore, the assessment shall be performed three times on each flexible line in a application where its length changes as follows: First, for the flexible line in its free length; second, for the flexible line in maximum thermal compression; third, for the flexible line in maximum thermal extension.

The analytical method in this document was developed only for metal bellows and flexhoses manufactured with formed annular convolutes, as shown in Figure 2. These are the most commonly used type in propellant systems. The analytical model was developed in reference 1. The equations in reference 1 were empirically derived from extensive testing and are the basis of this document.

<u>CAUTIONARY NOTE:</u> The analysis in this document was developed for normal flexible line installations. It does not allow for installations where unusual flow disturbances exist

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(except for elbows located upstream of the flexible line) or for multi-phase flows.

CAUTIONARY NOTE: This document is intended as a tool for analyzing only one portion of the total design of a flexible line. The engineer must consider all possible load sources other than flow-induced vibration when determining the total system life of the flexible line (see paragraph 3.0). The engineer must also consider other requirements (stability, pressure capability, etc.) not covered by this document in the design of a flexible line.

1.2 Excluded Flexible Line Assemblies

A. Instrumentation flexible lines.

B. Flexible lines with steady-state flow of less than one second duration.

C. Flexible lines with liners and sliding joints.

D. Components which do not fall into any of the following flight criticality categories:

I. Personnel hazard

II. Mission/vehicle loss

III. Launch delay

1.3 Media

Design analysis shall be repeated to verify the flexible line design integrity for all media imposed on the line, such as when a substitute medium is to be used in ground system checkout or other flow tests.

1.4 Design Criteria

There are two design criteria in which the flexible line (bellows and flexhose) shall be designed to meet. These are listed below:

1. The flexible line shall be designed to meet a theoretically infinite life, if its high cycle material curve exhibits a true endurance limit, for flow-induced vibration loads at its expected operating conditions. For a material not exhibiting a true endurance limit, an endurance limit as defined by each program shall be used.

2. The maximum operating flow velocity of the flexible line shall be limited per paragraph 2.6.

2.0 DESIGN ANALYSIS PROCEDURE

The procedure for analyzing a given bellows or flexhose configuration for susceptibility to fatigue failure

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from only flow-induced vibration loads consists of several different steps as follows:

For a Bellows:

Step 1. Calculate the natural frequencies for all vibration modes of the bellows: longitudinal modes and local convolute bending mode (see paragraph 2.1.1).

Step 2. Calculate the flow excitation velocity range for each mode of vibration (see paragraph 2.2).

Step 3. If the flow medium is a gas, calculate the first radial acoustic mode frequency and velocity (see paragraph 2.3).

Step 4. Calculate the flow-induced stress for each mode of vibration defined in step 1 and apply the appropriate uncertainty factors (see paragraph 2.4).

Step 5. Determine whether infinite life is achieved (see paragraph 2.5).

Step 6. Determine the maximum operating velocity limit (see paragraph 2.6).

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For a Flexhose:

Step 1. Calculate the natural frequencies for the three vibration modes of the flexhose: in-phase longitudinal mode, out-of-phase longitudinal mode, and local convolute bending mode (see paragraph 2.1.2).

Step 2. Calculate the flow excitation velocity range for each mode of vibration (see paragraph 2.2).

Step 3. If the flow medium is a gas, calculate the first radial acoustic mode frequency and velocity (see paragraph 2.3).

Step 4. Calculate the flow-induced stress for each mode of vibration defined in step 1 and apply the appropriate uncertainty factors (see paragraph 2.4).

Step 5. Determine whether infinite life is achieved (see paragraph 2.5).

Step 6. Determine the maximum operating velocity limit (see paragraph 2.6).

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2.1 Natural Frequency Calculation

2.1.1 Free Bellows

Consider the bellows structure represented by a lumped spring-mass mechanical model as shown in Figure 1. The pertinent bellows nomenclature used in the frequency calculation is given in Figure 2. All of the dimensions used in these calculations should be obtained by measuring the actual bellows being used. The user is advised that the as-built dimensions of a bellows can vary significantly from the specified drawing dimensions. As determined in reference 1, this can cause significant differences in the final results.

Step A. Calculate the elemental spring rate of one-half of a convolution, k, from the expression:

$$k=2N_cK_a \qquad (1)$$

where K_a is the overall bellows spring rate determined experimentally from a force-deflection test. For a new bellows assy., the user is required to employ experimental values obtained from a force-deflection test. For those bellows where a force-deflection test is not obtainable (i.e. a bellows already installed permanently in a line assy.), a

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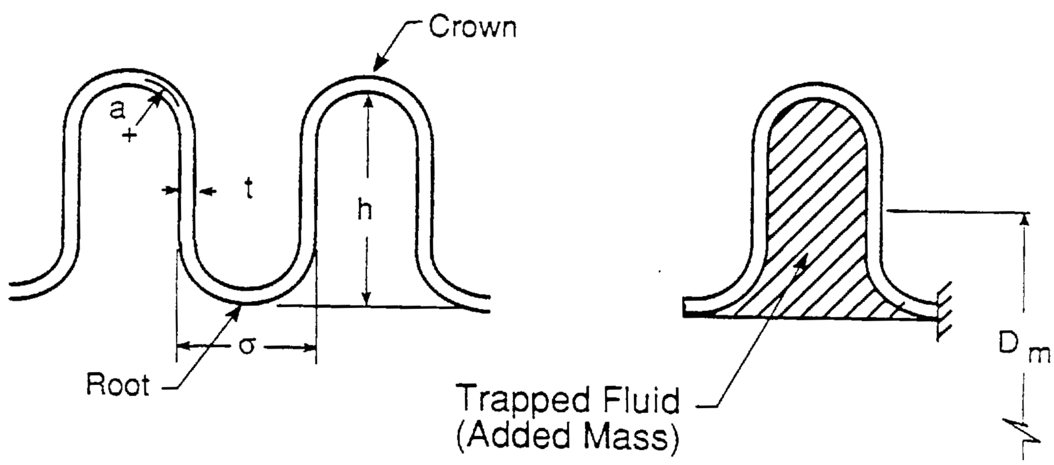
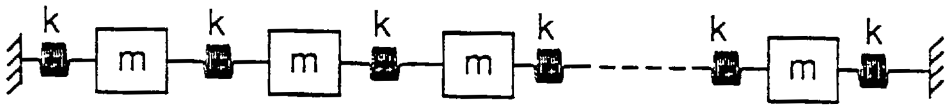
Convolution Nomenclature



Mechanical Model and Nomenclature



A mass (m) is assigned each convolution crown and root; the number of masses is $2N_C - 1$. The value of m is $m_m + m_f$ where

$$m_m = \frac{\pi \rho_m D_m t N_\rho \; [\pi a + (h-2a)]}{g}$$

The number of springs is $2N_C$ and

$$k = 2N_C K_A$$

where K_A is the overall bellows spring rate.

Figure 1. Lumped Spring-Mass Mechanical Model for Free Bellows

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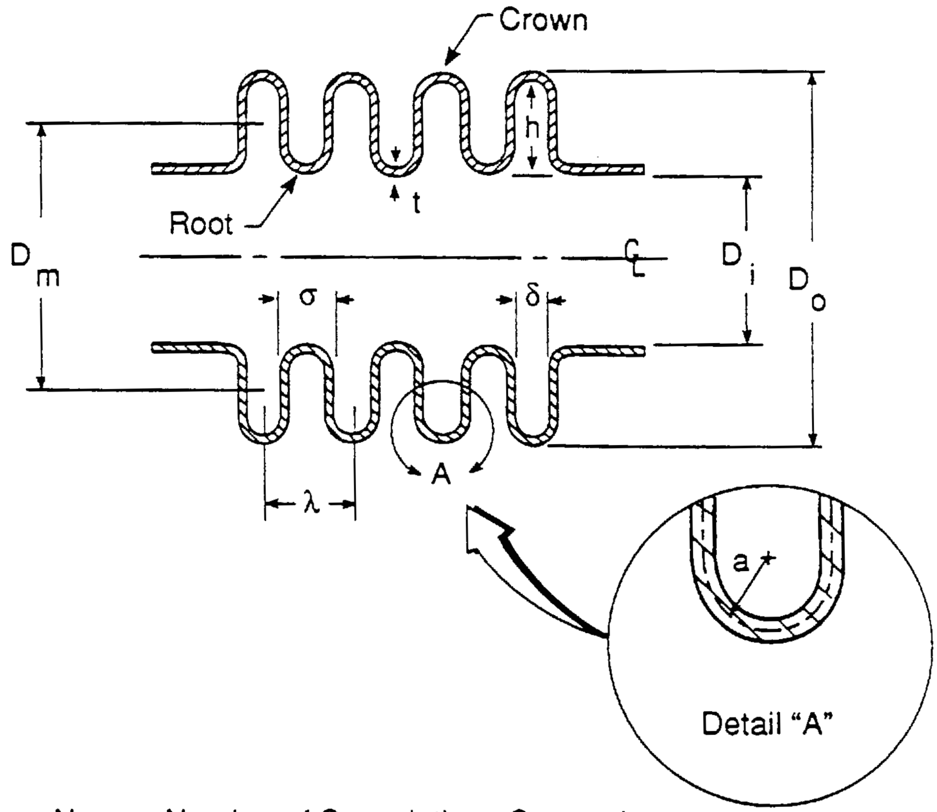

N_c = Number of Convolutions Counted from the Outside

N_p = Number of Plys

D_m = Mean Bellows Diameter

t = Wall Thickness (Thickness per Ply if Multi-Ply)

λ = Inside Convolute Pitch

σ = Inside Convolute Width

a = Mean Convolute Radius

h = Mean Inside Convolute Height

δ = Inside Convolute Gap

Figure 2. Bellows Nomenclature

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rough estimate of K_a may be made from the following expression:

$$K_a = D_m E (N_p / N_c)(t/h)^3 \qquad (2)$$

where E is Young's modulus for the bellows material at the operating temperature. If the bellows is designed to operate in the plastic range of the material then an adjusted value of E should be used in the calculations throughout this spec. One suggested method for adjusting E is discussed in paragraph 3.0.

Step B. Calculate the elemental metal mass, m_m, from the equation:

$$m_m = \frac{\pi \rho_m D_m t N_p [\pi a + h - 2a]}{g} \qquad (3)$$

where ρ_m = weight density of bellows material (lbf/in^3)

g = gravitational acceleration

a = mean convolute radius = $(\sigma - tN_p)/2$

h = mean inside convolute height

D_m = mean diameter of bellows

t = ply thickness

N_p = number of plys

Step C. Calculate the elemental fluid added mass, m_f, consisting of two types of loading, m_{f1} and m_{f2}, given as follows:

$$m_{f1} = \frac{\pi \rho_f D_m h(2a-tN_p)}{2g} \quad (4)$$

$$m_{f2} = \frac{\rho_f D_m h^3}{g\,\delta} \quad (5)$$

where ρ_f = weight density of fluid (lbf/in^3)

δ = inside convolute gap = $\lambda - \sigma$

Now, the total elemental fluid added mass in slugs is given by the empirical equation:

$$m_f = K_1 m_{f1} + K_2 m_{f2}(N/N_c) \quad (6)$$

where K_1 = 1.0 (non-dim)

K_2 = 0.68 (non-dim)

N = mode number = $1,2,3,\ldots 2N_c-1$

Step D. Calculate the dimensionless frequency factor, B_N, for each mode number N from the equation:

$$B_N = \{2[1+\cos(180(2N_c-N)/2N_c)]\}^{1/2} \quad (7)$$

Alternately, the dimensionless frequency factor may be obtained from Table 1 for certain values of N.

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Table 1. Dimensionless Frequency Factors B_N

MODE NUMBER N

N_C	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
1	1.414																								
2	0.765	1.414	1.845																						
3	0.520	1.000	1.414	1.732	1.930																				
4	0.390	0.765	1.111	1.414	1.663	1.848	1.962																		
5	0.314	0.618	0.908	1.176	1.414	1.618	1.782	1.902	1.975																
6	0.264	0.518	0.765	1.000	1.217	1.414	1.587	1.732	1.848	1.932	1.983														
7	0.226	0.445	0.661	0.868	1.064	1.247	1.414	1.563	1.693	1.802	1.888	1.950	1.987												
8	0.199	0.390	0.583	0.765	0.942	1.111	1.269	1.414	1.546	1.663	1.764	1.848	1.913	1.962	1.990										
9	0.174	0.347	0.518	0.684	0.845	1.000	1.147	1.285	1.414	1.532	1.638	1.732	1.812	1.879	1.931	1.969	1.992								
10	0.157	0.313	0.467	0.618	0.765	0.908	1.044	1.175	1.298	1.414	1.520	1.618	1.705	1.782	1.847	1.902	1.944	1.975	1.993						
11	0.142	0.285	0.425	0.563	0.699	0.831	0.958	1.081	1.198	1.309	1.414	1.511	1.601	1.682	1.755	1.819	1.873	1.918	1.954	1.979	1.994				
12	0.131	0.262	0.390	0.518	0.643	0.765	0.885	1.000	1.111	1.217	1.318	1.414	1.503	1.586	1.662	1.732	1.793	1.847	1.893	1.931	1.961	1.982	1.995		
13	0.121	0.241	0.361	0.479	0.595	0.709	0.821	0.929	1.034	1.136	1.233	1.326	1.414	1.497	1.574	1.645	1.711	1.770	1.823	1.870	1.909	1.941	1.967	1.985	1.996

NOTE: The Dimensionless Frequency Factors were Determined from the Equation

$$B_N = \sqrt{2\left\{1 + \cos\left[\frac{180(2N_c - N)}{2N_c}\right]\right\}}$$

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Step E. Calculate the reference frequency from the equation:

$$f_o = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \quad (8)$$

where f_o = reference frequency

k = elemental spring rate

$m = m_m + m_f$ = total elemental mass

<u>NOTE:</u> For a free bellows there are two different kinds of structural modes which may be flow-excited. They are the longitudinal modes and the local convolute bending mode. These are illustrated in Figure 3.

Step F. Calculate the true longitudinal mode frequencies for each mode number N from the equation:

$$f(N) = (f_o)\ (B_N) \quad (9)$$

where $f(N)$ = modal frequency (Hz)

Step G. Calculate the local convolute bending mode frequency from

$$f_{CB} = \frac{1}{2\pi}\sqrt{\frac{8k}{m_m + .68m_{f2}}} \quad (10)$$

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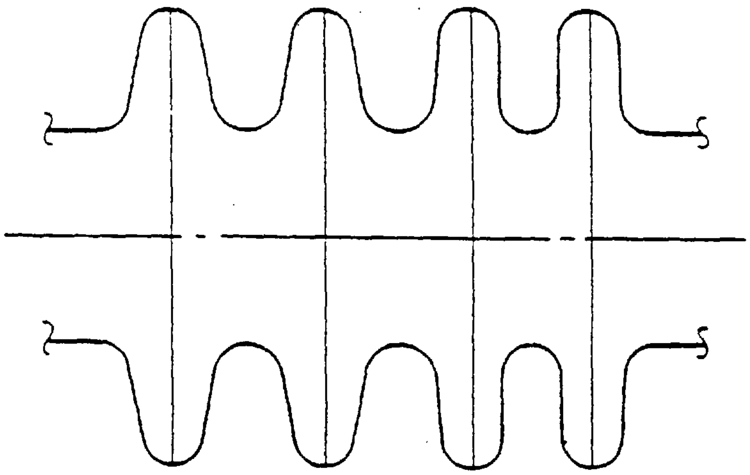
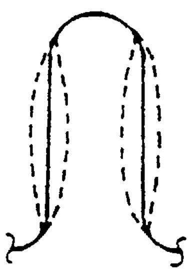


Axisymmetric Longitudinal Modes



Higher Order Local Convolute Bending Mode

Figure 3. Summary of Bellows Vibration Modes

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2.1.2 Flexhose

Consider the convoluted hose structure represented by the lumped spring-mass mechanical model as shown in Figure 4. Note that for a flexhose the value N_c=1 will be used.

Step A. Calculate the elemental spring rate of one-half of a convolution, k, from the expression:

$$k = 2K_f \qquad (11)$$

where K_f is the spring rate for one complete convolution (N_c=1) determined experimentally from a force-deflection test. Note that the overall spring rate obtained from test must be multiplied by the actual number of convolutes in the hose to obtain K_f. For a new flexhose assy., the user is required to employ experimental values obtained from a force-deflection test. For those flexhoses where a force-deflection test is not obtainable (i.e. a flexhose already installed permanently in a line assy.), a rough estimate of K_f may be made from the following expression:

$$K_f = D_m E N_p (t/h)^3 \qquad (12)$$

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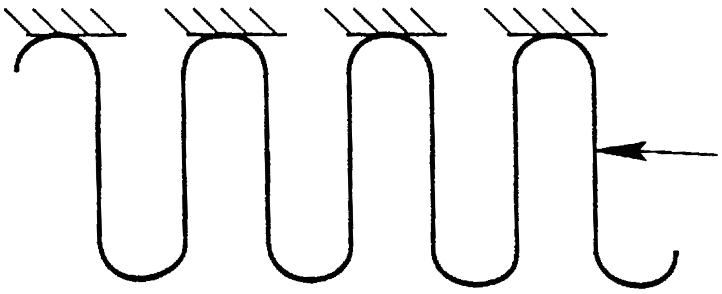

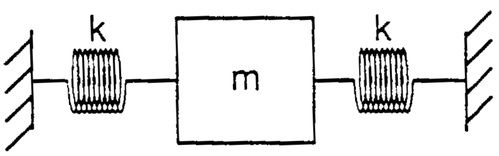

STRUCTURAL MODE FOR FLEXHOSE - With the bellows pressurized, longitudinal movement of the crown is restricted due to the braid. The root may move in-phase or out-of-phase with the adjacent convolute with a single degree of freedom. Therefore, the number of masses is one; which implies $N_c=1$; and $k=2K_f$.

Figure 4. Lumped Spring-Mass Mechanical Model for Flexhose

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Step B. Calculate the elemental metal mass using Equ. (3) as done previously.

Step C. Calculate the in-phase and out-of-phase longitudinal mode elemental fluid masses, m_{IP} & m_{OP}, respectively from the expressions:

$$m_{IP} = \frac{\pi \rho_f D_m h(2a-tNp)}{2g} \qquad (13)$$

$$m_{OP} = \frac{0.68 \rho_f D_m h^3}{g\,\delta} \qquad (14)$$

NOTE: For a flexhose there are, so far as is presently known, only three possible structural vibration modes which may be flow-excited. They are the in-phase and out-of-phase longitudinal modes, and the local convolute bending mode as illustrated in Fig. 5. This is true only if the braid is maintained in full contact with all convolute crowns. Should this not be the case, the engineer is cautioned that some sections of the hose may behave as free bellows and should be treated accordingly.

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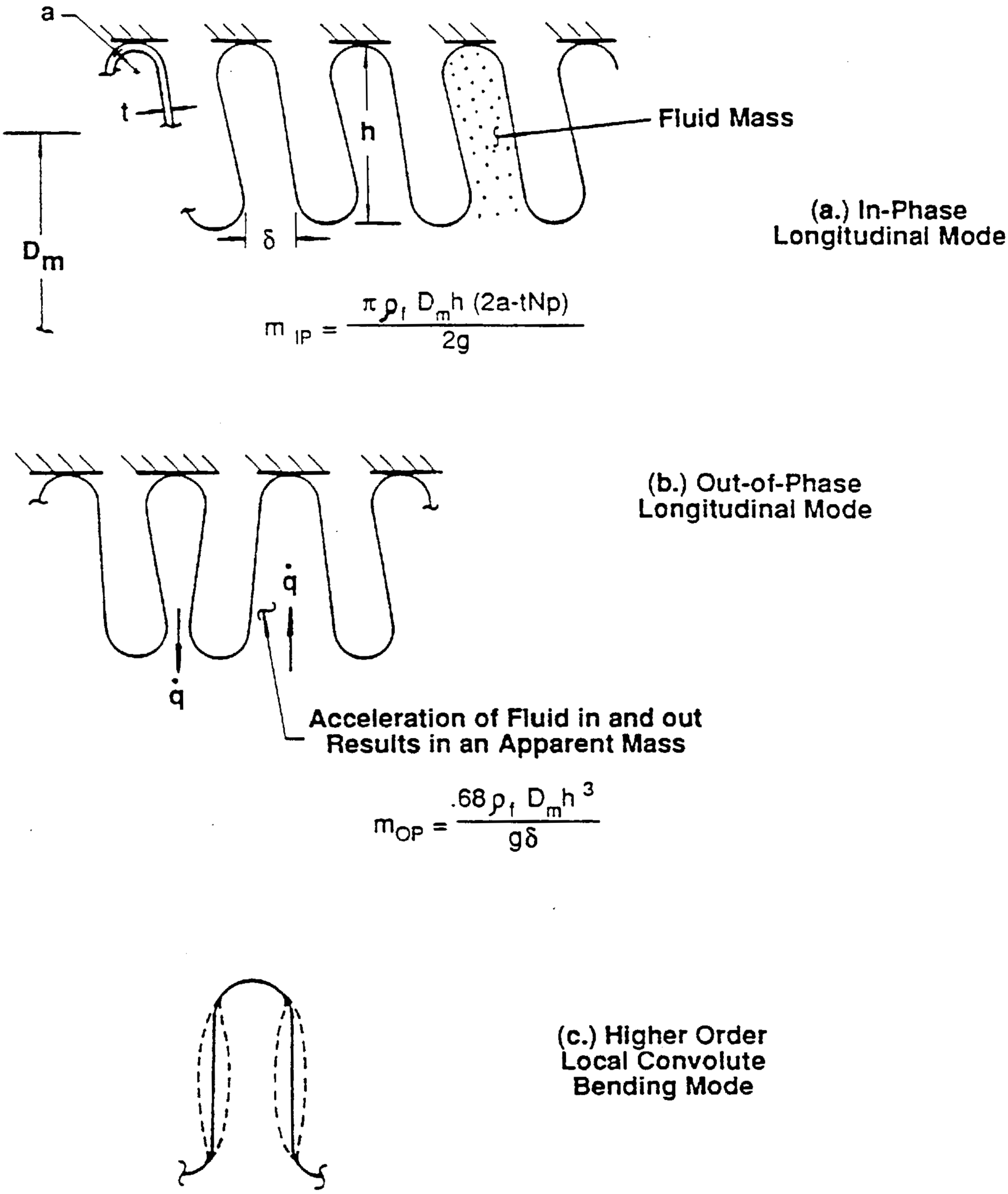

Figure 5. Summary of Flexible Hose Vibration Modes

Step D. Calculate the in-phase, out-of-phase, and convolute bending mode frequencies from the respective expressions:

$$f_{IP} = \frac{1}{2\pi}\sqrt{\frac{2k}{m_m + m_{IP}}} \tag{15}$$

$$f_{OP} = \frac{1}{2\pi}\sqrt{\frac{2k}{m_m + m_{OP}}} \tag{16}$$

$$f_{CB} = \frac{1}{2\pi}\sqrt{\frac{8k}{m_m + m_{OP}}} \tag{17}$$

As of now, there is no provision in the computer program for flexhose calculations. These must be done by hand.

2.2 Flow Excitation Range Calculation

Each bellows mode and flexhose mode may experience flow excitation over a fluid velocity range from a lower limit (V_{low}) to an upper limit (V_{up}) defined as

$$V_{low} = \frac{f(N)\sigma}{S_{\sigma u}} \tag{18}$$

$$V_{up} = \frac{f(N)\sigma}{S_{\sigma 1}} \tag{19}$$

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where σ = inside convolute width = $2a + tN_p$

$S_{\sigma u}$ = upper limit Strouhal number

$S_{\sigma l}$ = lower limit Strouhal number

$f(N)$ = each modal frequency calculated for a free bellows or flexhose

<u>For a free bellows:</u> Those longitudinal mode frequencies given in equ. (9) and the convolute bending mode frequency given in equ. (10).

<u>For a flexhose:</u> The in-phase, out-of-phase, and convolute bending mode frequencies given in equs. (15), (16), and (17) respectively.

It has been found that for most bellows and flexhose configurations, $S_{\sigma u} = 0.3$ and $S_{\sigma l} = 0.1$. The optimum or most severe excitation for each bellows and flexhose mode will occur at a critical velocity (V^*) related to the critical Strouhal number ($S_{\sigma c}$) as follows:

$$V^* = \frac{f(N)\sigma}{S_{\sigma c}} \qquad (20)$$

where $S_{\sigma c} = 0.2$

The possible flow excitation range of bellows and flexhoses may be predicted as follows:

(a) Calculate the lowest and highest bellows and flexhose excitation frequency for all longitudinal modes and the convolute bending mode as summarized in paragraphs 2.1.1 and 2.1.2.

(b) Calculate the limits of fluid velocity (V_{low} and V_{up}) corresponding to these two frequencies.

(c) Compare this flow-induced velocity range with the known operating range of the bellows. If an overlap of these ranges exist, then excitation may occur.

A graphical illustration of predicting the possible flow excitation range is given in Figure 6.

2.3 First Radial Acoustic Mode Resonance Calculation (Gas Medium Only)

For a bellows or flexhose whose internal flow medium is a gas, there can occur a radial acoustic resonance. This acoustic mode can occur in addition to the longitudinal modes

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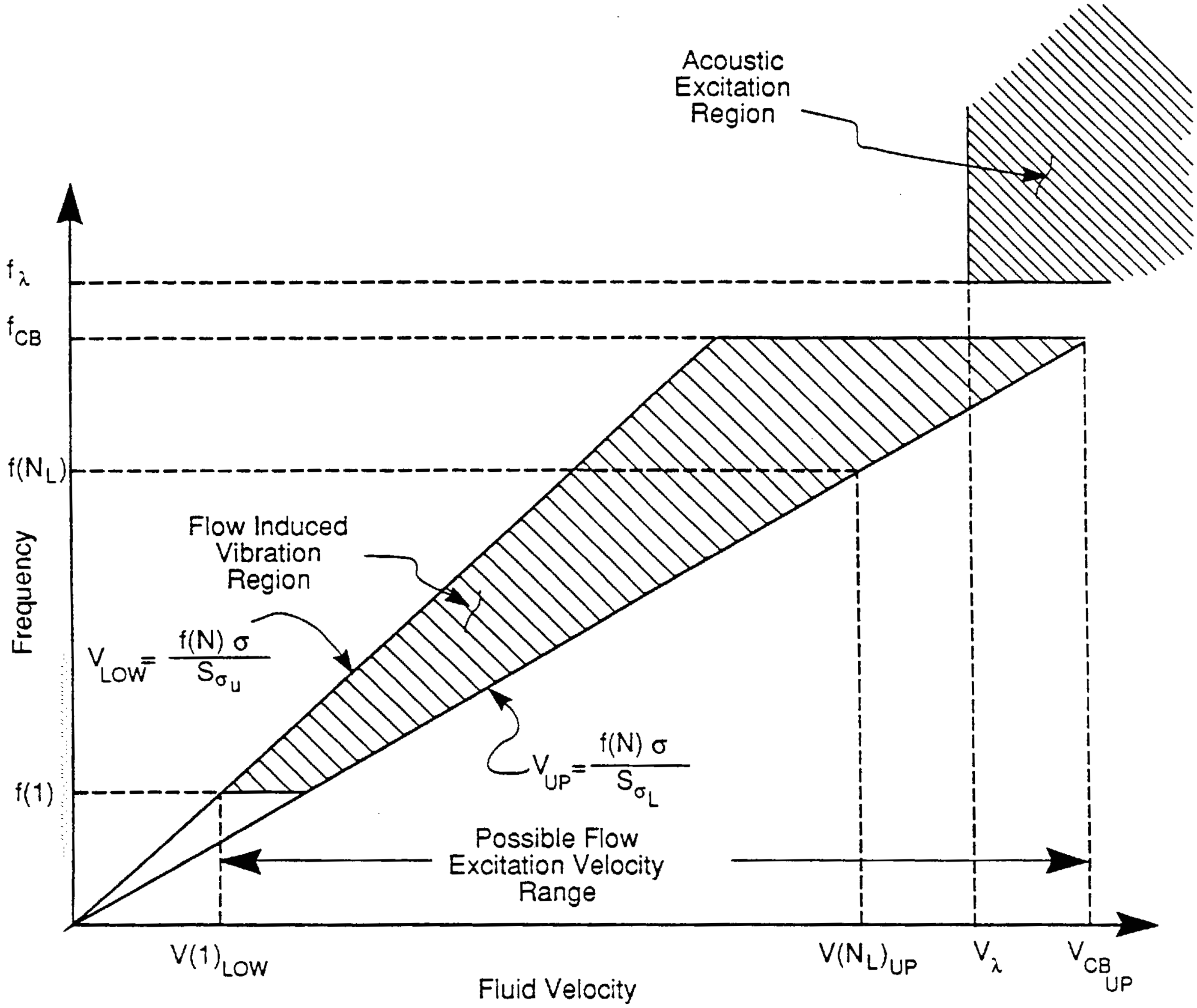

$f(1)$ = Frequency at longitudinal mode no. 1
$f(N_L)$ = Frequency at longitudinal mode no. $2N_c$-1
f_{CB} = Convolute bending mode frequency
f_λ = First radial acoustic mode frequency
$V(1)_{LOW}$ = Lower velocity for mode no. 1
$V(N_L)_{UP}$ = Upper velocity for mode no. $2N_c$-1
$V_{CB_{UP}}$ = Upper velocity for convolute bending mode
V_λ = First radial acoustic mode velocity

Note: f_λ can occur at any point in the frequency range. For the example shown here it is shown to fall above f_{CB} while depending on bellows and fluid parameters it might fall down in the longitudinal mode range.

Figure 6. Frequency Vs. Velocity Plot Indicating Flow Excitation Range

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and convolute bending mode. The frequency at which the first radial acoustic mode occurs is given by:

$$f_{\lambda} = \frac{(FNCO)(C_{\phi})}{2\pi r_i} \tag{21}$$

where C_{ϕ} is the speed of sound and is determined from fluid property data or if the fluid behaves approximately as an ideal gas then one can use the equation below.

$$C_{\phi} = \sqrt{\frac{\gamma(P+14.7)g}{\rho_f}}$$

P = fluid pressure

$$FNCO = 3.8-16.72(h/r_i)^2+13.67(h/r_i)^3 \quad \text{for } 0 \leq (h/r_i) < 0.4$$

$$FNCO = -.336+.935(h/r_i)^{-1} \quad \text{for } 0.4 \leq (h/r_i) \leq 1.0$$

where $r_i = D_i/2$

If the longitudinal mode or convolute bending mode frequencies are greater than or equal to the first radial acoustic mode frequency, then the flow-induced stress value (FIS) is multiplied by an acoustic factor of five (5) to account for acoustic amplification. Calculation of FIS is described in paragraph 2.4. If the longitudinal mode or convolute bending mode frequencies are less than the first radial acoustic mode frequency, then FIS is not adjusted by an acoustic factor.

The velocity at which the first radial acoustic mode occurs is given by

$$V_{\lambda} = \frac{f_{\lambda}\sigma}{S_{\sigma c}}$$

where $S_{\sigma c} = 0.2$

2.4 Flow-Induced Stress Calculation

In all velocity range overlap situations, flow-induced vibrations must be assumed to exist. Therefore, flow-induced stresses must be calculated in order to determine if a given bellows or flexhose configuration meets the design criteria of infinite life (see paragraph 1.4). Flow-induced stresses can be calculated from the following equation:

$$FIS = (EE)\left(\frac{C^{*}tP_D}{V' SSR\,\delta}\right)(E)(C_{NP})(C_E)\left(\frac{1}{N_P}\right) \quad (22)$$

where $EE = 1 + 0.1\left(\frac{400}{SSR}\right)^2$ (NON-DIM)

NOTE: The number 400 in the above equation is a reference specific spring rate having the units lbf/in^2.

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For a bellows: $SSR = \dfrac{K_a N_c}{D_m N_p}$

For a flexhose: $SSR = \dfrac{K_f N_c}{D_m N_p}$ and $N_c=1$

For all modes except the convolute bending mode use C^* equation below.

$$C^* = \frac{C_1}{C_2 + (V')^2} + \frac{C_3 |\sin(180V')|}{C_4 + (V')^2} + C_5 \quad \text{(NON-DIM)}$$

For the convolute bending mode use $C^* = 0.4$

$$V' = \frac{V^*}{V_c}$$

where for a bellows:

V^* = critical free stream velocity for a given longitudinal mode number N or for the convolute bending mode

V_c = the critical velocity for longitudinal mode $N=N_c$

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where for a flexhose:

V^* = critical free stream velocity for each of the three flexhose modes

V_c = the critical velocity for the flexhose out-of-phase mode with frequency f_{OP} given in equ. (16), so that

$$V_c = \frac{f_{OP}\,\sigma}{S_{\sigma c}}$$

where $S_{\sigma c} = 0.2$

$$P_D = \frac{\rho_f (V^*)^2}{2g}$$

$$\delta = \lambda - \sigma$$

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$$C_{NP} = \begin{cases} 1.0 & \text{for } N_p = 1 \\ 1.0 - \dfrac{C_6(\sigma/h)}{1.0+C_7(V')^2} & \text{for } N_p = 2,3,\ldots \end{cases}$$

$$C_E = \begin{cases} 1.0 & \text{For no elbow present upstream of bellows.} \\ 1.0 + \dfrac{4.7}{2.0+L/D} & \text{For elbow present upstream of bellows.} \end{cases}$$

where L = distance from elbow termination to the first bellows convolute

D = inside pipe diameter

The coefficients C_1, C_2,..., C_7 are non-dimensional empirical coefficients derived from the test data in reference 1 and have the following values:

$C_1 = 0.13$

$C_2 = 0.462$

$C_3 = 1.0$

$C_4 = 10.0$

$C_5 = 0.06$

$C_6 = 1.25$

$C_7 = 5.5$

2.4.1 Uncertainty Factors

A uncertainty factor (UF) is to be applied to the basic theoretical value of flow-induced stress (FIS) resulting in a corrected stress value given by

$$FISC = (FIS)(UF) \tag{23}$$

where UF is determined as follows:

(a) For a free bellows if measured spring rate is used, UF $\geq$ 1.5

(b) For a free bellows where spring rate is estimated from equ. (2), UF $\geq$ 2.0

(c) For a flexhose where measured spring rate is used, UF $\geq$ 2.0

(d) For a flexhose where spring rate is estimated from equ. (12), UF $\geq$ 2.5

(e) For a bellows or flexhose with radial acoustic resonance, multiply the above factors (a) through (d) by 1.5.

Note that these uncertainty factors are applied only to account for uncertainties in the analysis and data base and should not be confused with programmatic safety factors.

In addition to the above uncertainty factors, (a) through (e), an acoustic factor is to be applied in the case

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of radial acoustic resonance, as discussed already in paragraph 2.3. If any modal frequency for a given bellows or flexhose mode is greater than or equal to the first radial acoustic mode frequency (f_{λ}), then an acoustic factor of five (5) must also be applied to the flow-induced stress value (FIS). If the modal frequency is less than the first radial acoustic mode frequency, then FIS is not adjusted by an acoustic factor.

2.5 <u>Bellows and Flexhose Fatigue Assessment</u>

After calculating the flow-induced stresses and applying the appropriate uncertainty factors, an assessment of the fatigue life must now be made. The fatigue life criterion is that the flexible line shall be designed to meet a theoretically infinite life for flow-induced vibration loads at its expected operating conditions (para. 1.4). For the assessment a comparison should be made between FISC and the endurance limit of the flexible line material.

NOTE: For a material not exhibiting a true endurance limit, an endurance limit as defined by each program shall be used.

If FISC is less than the endurance limit then infinite life is achieved. If FISC is greater than the endurance limit

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then infinite life cannot be met. <u>This fatigue assessment assumes the mean stress of the flexible line is zero or negligible.</u> A system level analysis to predict the overall life of the flexible line must be performed, although not required by this document. This system level analysis is discussed in paragraph 3.0.

If infinite life cannot be achieved through this fatigue assessment, then a redesign and reanalysis is necessary or the maximum flow velocity must be restricted in order to meet infinite life. If it is determined that a bellows redesign is necessary and if no other geometrical configurations are available or possible, then bellows liners may be required. Liners isolate the convolutes from flow impingement, thereby eliminating flow-induced vibration occurence. However, a weight and cost penalty may be associated with the installation of liners. Liners should be designed to minimize pressure differential (delta P), and where there is reverse flow, two-piece liners should be used.

2.6 <u>Bellows and Flexhose Operating Velocity Assessment</u>

Once it has been established that the bellows and flexhose will have infinite life, it is still necessary to limit the maximum operating velocity of the line. This is necessary because of the uncertanties beyond the last mode predicted. Depending on the case, the maximum operating velocity of the bellows and flexhose shall be limited as follows:

Case A: For liquid flow in a bellows and flexhose, where infinite life is predicted for all longitudinal modes and the convolute bending mode, the maximum operating velocity shall be limited to the upper limit (V_{up}) of the convolute bending mode.

Case B: For gas flow in a bellows and flexhose, where infinite life is predicted for all modes (longitudinal and convolute bending) and the first radial acoustic mode velocity (V_{λ}) is less than the upper limit (V_{up}) of the convolute bending mode, the maximum operating velocity shall be limited to (V_{up}) of the convolute bending mode.

Case C: If in Case B, the first radial acoustic mode velocity (V_{λ}) was greater than the upper limit (V_{up}) of the convolute bending mode, then the maximum operating velocity shall be limited to the lesser of V_{up} of the convolute bending mode or 80% of V_{λ}.

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Case D: For liquid or gas flows in a bellows and flexhose, where infinite life cannot be met for all of its modes, the maximum operating velocity shall be limited to less than the lower limit (V_{low}) of the mode that first indicates finite life.

3.0 SYSTEM LEVEL ANALYSIS OF A FLEXIBLE LINE

Although the requirements of this document deal strictly with assessing the fatigue life of flexible lines from flow-induced vibration loading, the designer must realize that flow-induced vibration is only one source of a wide spectrum of loads imposed on a flexible line. Strength and fatigue analyses which include all of the load sources imposed on a flexible line must be performed. The flowchart in Figure 7 shows a general path one might follow, and some of the load sources which must be considered, in performing the analyses of a flexible line.

This section presents a few comments, guidelines, and recommendations on performing a system level analysis of a flexible line. Even though a system level analysis is not within the scope of this document and also not a requirement of this document, it should be performed sometime in the

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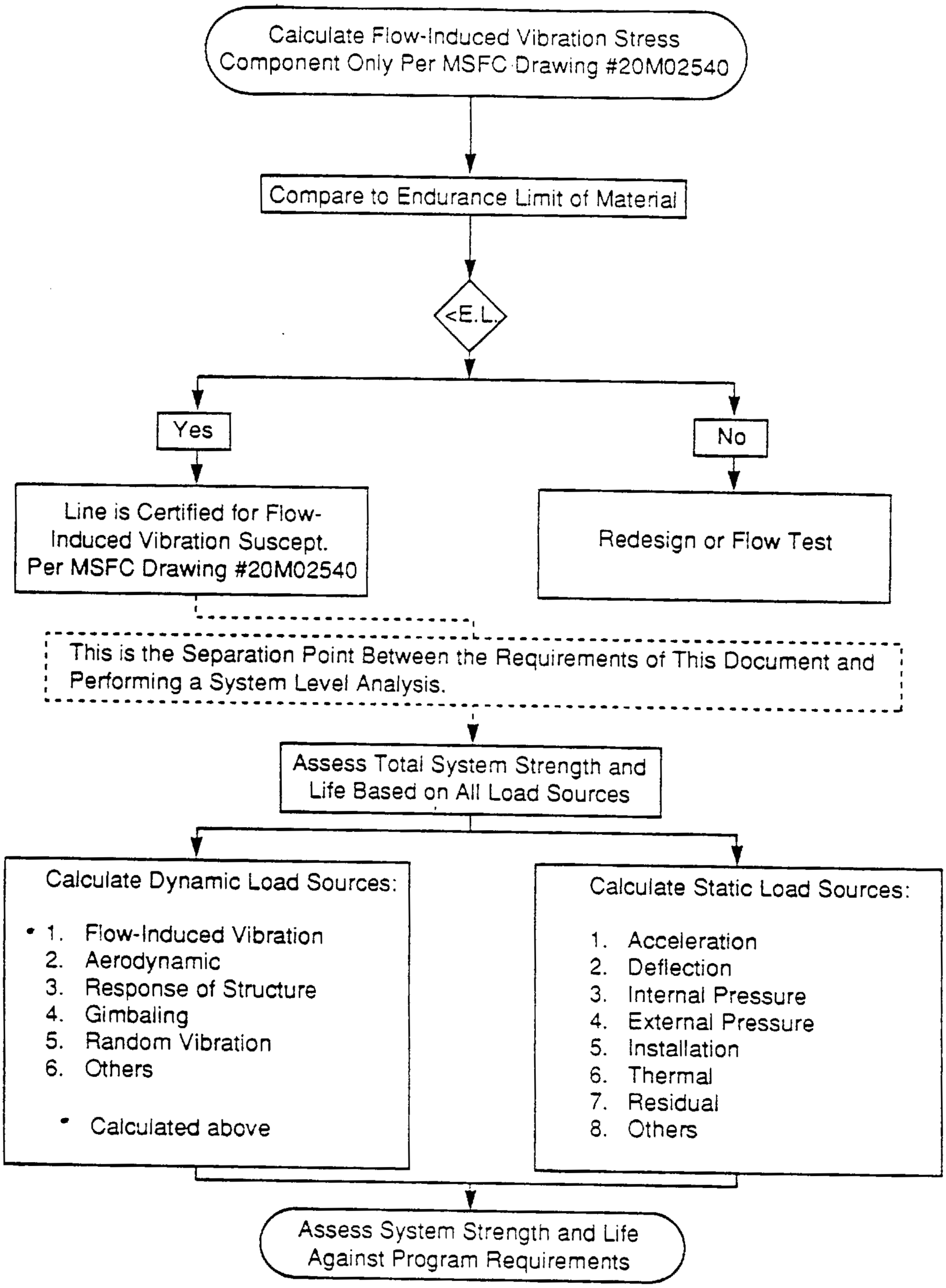

Figure 7. Flow Chart for System Level Analysis

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design process. All flexible line analysis should be reviewed and accepted by the governing agency.

1. Equations for the static stresses resulting from deflection and static pressure in a flexible line operating in the elastic material range may be found in references 2, 3, 4, and 5.

2. Flexible lines, by definition, must be flexible and capable of accommodating deflections across its length. Because of this, many flexible lines operate in the non-linear (plastic) material range. This phenomenon increases the difficulty of the analysis and requires that good engineering judgement and analysis procedures be used to assure that all loading effects are accounted for. The following are some items to be considered in performing a plastic analysis:

A. A computer code capable of including large displacement and non-linear material effects will be needed. This is necessary to determine the stress field along the meridian and through the wall thickness of the flexible line. Plastic behavior of the material would be expected in the inner radius of the convolutes at the root and crown.

B. Gross yielding of the material should be avoided. Yielding should not occur more than 25% of the way through the wall thickness.

C. Margins of safety may need to be calculated based on strain capability. Maximum strains must include the effects of all loading, both static and dynamic.

D. Strains induced by static loading; e.g., deflections, pressures, etc., may be calculated by use of finite difference or finite element computer codes capable of handling large deflection and non-linear material effects. Dynamic loading effects; e.g., flow-induced vibration stresses, however, cannot be modeled directly with these methods and are usually known as loads across the flexible line or stresses in the flexible line. Several effects must be taken into consideration when calculating strains from these loads and stresses. 1. The spring rate of the flexible line changes as the line is deflected. The deflection due to the dynamic load should be calculated using the spring rate consistent with the configuration of the line when the loading is applied. The resultant strains can then be calculated using the methods employed for the static loading. 2. The material modulus of elasticity will vary along the meridian

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and through the wall thickness of the flexible line, depending on if the material at that point has yielded, and the amount of yielding experienced. Therefore, the modulus of elasticity needs to be adjusted. This must be considered when calculating strains due to dynamic stresses. One factor which may be used to adjust the elastic modulus is the ratio of the spring rate of the flexible line consistent with the configuration of the line when the stresses occur, versus the spring rate of the flexible line in the undeflected condition.

E. The change in material modulus when yielding occurs will affect the response of the flexible line to the flow. The calculation of flow-induced stresses should be repeated with an adjusted modulus if the line is found to have yielded. The method previously discussed can be used for adjusting the modulus of elasticity.

3. Fatigue analysis of the line must include effects of creep, low cycle, and high cycle loading. Life fractions for the creep, low cycle, and high cycle fatigue must include the required life factors and are additive. Their sum must be less than or equal to one.

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These are just a few of the issues which must be considered when performing a system level analysis of a flexible line. The analyst should assure that all loading conditions are accounted for in the analysis. The analysis should also be accompanied by a test program which simulates the operating conditions of the flexible line as closely as possible. The analysis and test program should be approved by the governing agency of the project.

4.0 FLOW TESTING

When flow testing of a bellows or flexhose is necessary, it shall be conducted in accordance with MSFC-SPEC-626 and must demonstrate a safety factor of four (4) on life.

5.0 COMPUTER PROGRAM

A computer program for conducting flow-induced vibration analysis of only a free bellows is included in Appendix C. This program calculates the frequency, flow excitation range, and flow-induced stresses for each longitudinal mode and the convolute bending mode for a free bellows. This program applies for both liquid and gas flows through the bellows. This program also calculates the first

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radial acoustic mode resonance for gas flows. The program does not conduct flow-induced vibration analysis for a flexhose and does not calculate any static stresses. These flexhose and static stress analyses have to be done by hand.

Also given in Appendix C is the input data file format along with two examples. The corresponding output files for the two examples are also given. These two examples are the same as those presented in Appendix B.

6.0 EXAMPLE PROBLEMS

There are three examples of hand calculations for flow-induced stresses given in Appendix B.

The three examples are:

Example 1.1 - - - Liquid flow through a bellows

Example 1.2 - - - Gas flow through a bellows

Example 2.0 - - - Gas flow through a flexhose

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REFERENCES

1. Tygielski, P.J., Smyly, H.M., Gerlach, C.R., "Bellows Flow-Induced Vibrations," NASA Technical Memorandum, NASA TM-82556, George C. Marshall Space Flight Center, Huntsville, Alabama, October 1983.

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4. Anderson, W.F., 1964, Analysis of Stresses in Bellows, Part I and Part II Design Criteria and Test Results, Atomics International, Report No. NAA-SR-4527, Canoga Park, California.

5. Standards of the Expansion Joint Manufacturers Association, Inc., 5th Edition, White Plains, New York, 1980.

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APPENDIX A

SYMBOLS AND DEFINITIONS

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SYMBOLS

a	= Mean convolute radius = $(\sigma - tN_p)/2$
B_N	= Dimensionless frequency factor
C_E	= Elbow factor (non-dim)
C_{NP}	= Damping modifier coefficient (non-dim)
C^*	= Force and damping coefficient (non-dim)
C_ϕ	= Speed of sound
D_i	= Inside diameter of flexible line
D_m	= Mean diameter of flexible line = $(D_i+D_o)/2$
D_o	= Outside diameter of flexible line
E	= Young's modulus of elasticity
f_λ	= First radial acoustic mode frequency
f(N)	= Modal frequency
f_{IP}	= Flexhose in-phase longitudinal mode frequency
f_{OP}	= Flexhose out-of-phase longitudinal mode frequency
f_{CB}	= Convolute bending mode frequency
f_c	= Critical frequency for mode $N=N_c$
f_o	= Reference frequency
FNCO	= First radial acoustic mode frequency number (non-dim)
FIS	= Flow-induced stress
FISC	= Flow-induced stress with uncertainty factor

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SYMBOLS (Cont.)

g = Gravitational acceleration

h = Mean inside convolute height = $[(D_o-D_i)/2]-tN_p$

k = Elemental spring rate of one-half of a convolution

K_a = Overall bellows spring rate

K_f = Flexhose spring rate for one complete convolution

m = Total elemental mass

m_m = Elemental metal mass

m_f = Total elemental fluid added mass

m_{f1} = Fluid added mass

m_{f2} = Fluid added mass

m_{IP} = Flexhose in-phase elemental fluid mass

m_{OP} = Flexhose out-of-phase elemental fluid mass

N = Mode number $(1,2,3...2N_c-1)$

N_c = Number of convolutes counted from the outside

N_p = Number of plys

P_D = Free stream dynamic pressure

P = Fluid pressure

r_i = Inside flexible line radius = $D_i/2$

$S_{\sigma l}$ = Lower Strouhal number (non-dim)

$S_{\sigma u}$ = Upper Strouhal number (non-dim)

$S_{\sigma c}$ = Critical Strouhal number (non-dim)

SSR = Specific spring rate

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SYMBOLS (Cont.)

S_{EL} = Endurance limit of flexible line material

t = Ply thickness

UF = Uncertainty factor

V_{low} = Lower limit velocity for mode N

V^* = Critical velocity for mode N

V_{up} = Upper limit velocity for mode N

V_c = Critical velocity for mode $N=N_c$

V' = Normalized velocity parameter = V^*/V_c (non-dim)

V_λ = First radial acoustic mode velocity

γ = Specific heat ratio for the gas = C_p/C_V (non-dim)

σ = Inside convolute width = $2a + tN_p$

δ = Inside convolute gap = $\lambda - \sigma$

λ = Inside convolute pitch

ρ_f = Weight density of fluid (lbf/in^3)

ρ_m = Weight density of flexible line matl. (lbf/in^3)

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DEFINITIONS

Angulation - Angular deflection imposed on a flexible line.

Axial deflection - Elongation or compression of a flexible line along its longitudinal axis.

Flexhose - A flexible metal hose where convolutes are partially restrained at the crown by wire braid.

Flexible line - Metal bellows or flexhose assembly that joins two duct sessions and permits relative motion between the ducts in one or more planes.

Free bellows - Where convolutes have unrestricted movement when exposed to fluid flow impingement.

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APPENDIX B

FLOW-INDUCED VIBRATION EXAMPLE PROBLEMS

FOR BELLOWS AND FLEXHOSE

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1.0 BELLOWS EXAMPLE PROBLEMS

1.1 Liquid Medium Example

Given: H_2O flowing through a 3 inch 321 stainless steel bellows at 68 ^oF and at 35 psig with an elbow 4 inches from the first convolute.

BELLOWS PARAMETERS

Inside convolute width, σ = 0.095 in.

Inside convolute pitch, λ = 0.148 in.

Mean inside convolute height, h = 0.325 in.

Ply thickness, t = 0.007 in.

Inside diameter, D_i = 3.00 in.

Outside diameter, D_o = 3.69 in.

Number of convolutes, N_c = 16

Number of plys, N_p = 3

Young's modulus, E = 29.0E+06 psi

Material weight density, ρ_m = 0.286 lbf/cu. in.

Problem: Assess the fatigue life from flow-induced vibration loads for the first longitudinal mode N=1 and the longitudinal mode $N=N_c$.

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CALCULATION PROCEDURE:

I. Frequency Calculation for Longitudinal Mode N=1.

1. $K_a = D_m E(N_p/N_c)(t/h)^3$

$K_a = 3.345(29E{+}06)(3/16)(.007/.325)^3$

$K_a = 181.735$ lbf/in

$k = 2N_cK_a = 2(16)(181.735) = 5815.52$ lbf/in

2. $m = m_m + m_f$

$$m_m = \frac{\pi \rho_m t N_p D_m[\pi a + h - 2a]}{g}$$

$a = (\sigma - tN_p)/2 = [.095-.007(3)]/2 = 0.037$ in.

$$m_m = \frac{\pi(.286)(.007)(3)(3.345)[\pi(.037)+.325-2(.037)]}{32.174}$$

$m_m = 7.20E{-}04$ slugs

$m_f = K_1 m_{f1} + K_2 m_{f2}(N/N_c)$

$$m_{f1} = \frac{\pi \rho_f D_m h(2a - tN_p)}{2g}$$

$$m_{f1} = \frac{\pi(62.4/1728)(3.345)(.325)[2(.037)-.007(3)]}{2(32.174)}$$

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$$m_{f1} = 1.02E\text{-}04 \text{ slugs}$$

$$m_{f2} = \frac{\rho_f D_m h^3}{g\ \delta} = \frac{(62.4/1728)(3.345)(.325)^3}{32.174(.148-.095)}$$

$$m_{f2} = 2.43E\text{-}03 \text{ slugs}$$

$$m_f = 1.0(1.02E\text{-}04) + 0.68(2.43E\text{-}03)(1/16)$$

$$m_f = 2.05E\text{-}04 \text{ slugs}$$

$$m = m_m + m_f = 7.20E\text{-}04 + 2.05E\text{-}04$$

$$m = 9.25E\text{-}04 \text{ slugs}$$

*3. $B_N = \{2[1+\cos(180(2N_c-N)/2N_c)]\}^{1/2}$

for N=1, $B_1 = \{2[1+\cos(180(32-1)/32)]\}^{1/2}$

$$B_1 = 0.0981$$

4. $f_o = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{12(5815.52)}{9.25E\text{-}04}} = 1382.40 \text{ Hz}$

5. $f(1) = (f_o)(B_1)$

$f(1) = (1382.40)(.0981) = 135.61 \text{ Hz}$

*If calculating cos in degrees, use 180; if calculating cos in radians, use π.

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II. Velocity Range Calculation for Longitudinal Mode N=1.

$$V(N,i) = \frac{f(N)\sigma}{(S_{\sigma i})} \quad \text{where } N = 1,2,3\ldots,\ 2N_c-1,\quad i = 1,2,3$$

$$S_{\sigma 1} = S_{\sigma u} = 0.3$$

$$S_{\sigma 2} = S_{\sigma c} = 0.2$$

$$S_{\sigma 3} = S_{\sigma l} = 0.1$$

$$V_{low} = V(1,1) = \frac{f(1)\sigma}{(S_{\sigma u})} = \frac{135.61(.095)}{12(0.3)} = 3.58 \text{ fps}$$

$$V^{*} = V(1,2) = \frac{f(1)\sigma}{(S_{\sigma c})} = \frac{135.61(.095)}{12(0.2)} = 5.37 \text{ fps}$$

$$V_{up} = V(1,3) = \frac{f(1)\sigma}{(S_{\sigma l})} = \frac{135.61(.095)}{12(0.1)} = 10.74 \text{ fps}$$

III. Flow-Induced Stress Calculation for Longitudinal Mode N=1

1. Critical frequency (f_c) at $N=N_c$

$$m_f = 1.0(1.02E-04) + 0.68(2.43E-03)(16/16)$$

$$m_f = 1.75E-03 \text{ slugs}$$

$$m = m_m + m_f = 7.20E-04 + 1.75E-03$$

$$m = 2.47E-03 \text{ slugs}$$

$$@\ N = N_c,\ B_{16} = \sqrt{2}$$

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$$f_c = (f_o)(B_{16})$$

$$f_c = \frac{1}{2\pi}\sqrt{\frac{12(5815.52)}{2.47E-03}}\left(\sqrt{2}\right) = 1196.4 \text{ Hz}$$

2. Critical velocity (V_c) at $N=N_c$

$$V_c = \frac{f_c\,\sigma}{(S_{\sigma c})} = \frac{1196.4(.095)}{12(0.2)} = 47.36 \text{ fps}$$

3. $V' = V^*/V_c = \frac{5.37}{47.36} = 0.113$

4. $SSR = \frac{K_a N_c}{D_m N_p} = \frac{181.735(16)}{3.345(3)} = 289.762 \text{ lbf/in}^2$

*5. $C_{NP} = 1.0 - \frac{C_6\ (\sigma/h)}{1 + C_7\ (V')^2}$

$$C_{NP} = 1.0 - \frac{1.25(.095/.325)}{1 + 5.5(.113)^2} = .659$$

$$C^* = \frac{C_1}{C_2 + (V')^2} + \frac{C_3|\sin(180V')|}{C_4 + (V')^2} + C_5$$

$$C^* = \frac{0.13}{0.462 + (.113)^2} + \frac{1.0|\sin[180(.113)]|}{10.0 + (.113)^2} + 0.06$$

$$C^* = 0.369$$

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6. $$P_D = \frac{\rho_f (V^*)^2}{2g} = \frac{(62.4/1728)(5.37)^2(12)}{2(32.174)} = 0.194 \text{ psi}$$

7. $$DD = \frac{C^* t P_D}{V' SSR\ \delta} = \frac{0.369(.007)(.194)}{0.113(289.762)(.053)} = 2.89E-04$$

*If calculating sin in degrees, use 180; if calculating sin in radians, use π.

8. $$EE = 1.0 + 0.1 \left(\frac{400}{SSR}\right)^2 = 1.0 + 0.1 \left(\frac{400}{289.762}\right)^2$$

$$EE = 1.191$$

9. $$C_E = 1.0 + \frac{4.7}{2 + L/D} = 1.0 + \frac{4.7}{2 + 4/3} = 2.41$$

10. $$FIS = \frac{(EE)\ (DD)\ (E)\ (C_{NP})\ (C_E)}{N_p}$$

11. $$FIS = \frac{1.191(2.89E-04)(29E+06)(.659)(2.41)}{3}$$

$$FIS = 5{,}284 \text{ psi for longitudinal mode } N=1$$

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Following the same procedure as above the value of FIS for the $N=N_c$ mode is determined to be

$$FIS = 26,873 \text{ psi for longitudinal mode } N=N_c$$

12. Uncertainty Factor:

The spring rate was estimated using equ. (2) therefore, the uncertainty factor is

$$UF = 2.0$$

and the corrected predicted flow-induced stress for longitudinal modes N=1 and $N=N_c$ is:

$$FISC = (UF)(FIS)$$

$$FISC = (2.0)(5,284) = 10,568 \text{ psi for long. mode } N=1$$

$$FISC = (2.0)(26,873) = 53,747 \text{ psi for long. mode } N=N_c$$

IV. Fatigue Assessment

From the results above

$$FISC = 10,568 \text{ psi for longitudinal mode } N=1$$

$$FISC = 53,747 \text{ psi for longitudinal mode } N=N_c$$

For 321 steel at 68°F the endurance limit is

$$S_{EL} = 26,500 \text{ psi}$$

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Conclusion:

For longitudinal mode N=1 FISC is less than S_{EL}, therefore, infinite life is predicted. For longitudinal mode $N=N_c$ FISC is greater than S_{EL}, therefore, finite life is indicated and the bellows must be redesigned if operated to a velocity capable of exciting this mode. Repeating this analysis for each mode lower than $N=N_c$ could be performed to find the mode and corresponding maximum velocity the bellows can be operated and still achieve infinite life.

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1.2 Gaseous Medium Example

Given: GN_2 flowing through an 8 inch 21-6-9 steel bellows at -200 ^oF and at 39.3 psig with no elbow upstream.

BELLOWS PARAMETERS

Inside convolute width, σ = 0.400 in.

Inside convolute pitch, λ = 0.726 in.

Mean inside convolute height, h = 1.25 in.

Ply thickness, t = .037 in.

Inside diameter, D_i = 8.00 in.

Outside diameter, D_o = 10.574 in.

Number of convolutes, N_c = 7

Number of plys, N_p = 1

Young's modulus, E = 28.5E+06 psi

Material weight density, ρ_m = 0.282 lbf/cu. in.

Problem: Determine the maximum safe flow velocity which will result in a predicted infinite life from flow-induced vibration loads.

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CALCULATION PROCEDURE:

I. Frequency Calculation for Longitudinal Mode N=1.

1. $K_a = D_m E(N_p/N_c)(t/h)^3$

$$K_a = \left(\frac{8.00+10.574}{2}\right)\left(28.5E+06\right)\left(\frac{1}{7}\right)\left(\frac{.037}{1.25}\right)^3$$

K_a = 980.61 lbf/in

$k = 2N_c K_a = 2(7)(980.61) = 13{,}728.54$ lbf/in

2. $m = m_m + m_f$

$$m_m = \frac{\pi \rho_m t N_p D_m [\pi a + h - 2a]}{g}$$

$a = (\sigma - tN_p)/2 = [.400-.037(1)]/2 = 0.182$ in

$$m_m = \frac{\pi(.282)(.037)(1)(9.287)[\pi(.182)+1.25-2(.182)]}{32.174}$$

m_m = 137.93E-04 slugs

$m_f = K_1 m_{f1} + K_2 m_{f2}(N/N_c)$

$$m_{f1} = \frac{\pi \rho_f D_m h(2a-tN_p)}{2g}$$

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where $\rho_f = \rho_{ref}(P/P_{ref})(T_{ref}/T)\ (Z_{ref}/Z)$

P = gas pressure, psia

P_{ref} = reference pressure = 14.7 psia

T = gas temperature, ^oR

Z = gas compressibility factor (non-dim)

T_{ref} = reference temperature = 528^oR

Z_{ref} = 1.0 = gas compressibility factor at reference condition for an ideal gas (non-dim)

ρ_{ref} = reference density (lbf/ft^3) =

$P_{ref}/RT_{ref}Z_{ref}$ from gas law

$$\rho_{ref} = \frac{14.7(144)}{54.92(528)(1.0)} = 0.073\ lbf/ft^3$$

$$\rho_f = \left(\frac{0.073}{1728}\right)\left(\frac{39.3+14.7}{14.7}\right)\left(\frac{528}{-200+460}\right)\left(\frac{1.0}{.982}\right)$$

$$\rho_f = 3.21E-04\ lbf/in^3$$

$$m_{f1} = \frac{\pi(3.21E-04)(9.287)(1.25)[2(.182)-.037(1)]}{2(32.174)}$$

$$m_{f1} = 5.95E-05\ slugs$$

$$m_{f2} = \frac{\rho_f D_m h^3}{g\,\delta} = \frac{3.21E-04(9.287)(1.25)^3}{32.174(.726-.400)}$$

$$m_{f2} = 5.55E-04\ slugs$$

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$$m_f = 1.0(5.95E-05) + 0.68(5.55E-04)(1/7)$$

$$m_f = 1.13E-04 \text{ slugs}$$

$$m = m_m + m_f = 137.93E-04 + 1.13E-04$$

$$m = 13.91E-03 \text{ slugs}$$

*3. $B_N = \{2[1+\cos(180(2N_c-N)/2N_c)]\}^{1/2}$

for N=1, $B_1 = \{2[1+\cos(180(14-1)/14)]\}^{1/2}$

$$B_1 = 0.2239$$

4. $f_o = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{12(13728.54)}{13.91E-03}} = 547.72 \text{ Hz}$

5. $f(1) = (f_o)(B_1) = 547.72(0.2239) = 122.63 \text{ Hz}$

*If calculating cos in degrees, use 180; if calculating cos in radians, use π.

II. Velocity Range Calculation for Longitudinal Mode N=1

$$V_{low} = V(1,1) = \frac{f(1)\sigma}{S_{\sigma u}} = \frac{122.63(.400)}{12(0.3)} = 13.63 \text{ fps}$$

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$$V^* = V(1,2) = \frac{f(1)\sigma}{S_{\sigma c}} = \frac{122.63(.400)}{12(0.2)} = 20.44 \text{ fps}$$

$$V_{up} = V(1,3) = \frac{f(1)\sigma}{S_{\sigma 1}} = \frac{122.63(.400)}{12(0.1)} = 40.88 \text{ fps}$$

III. First Radial Acoustic Mode Resonance Calculation

1. $h/r_i = 1.25/(8.00/2) = .3125$

2. $FNCO = 3.8 - 16.72(h/r_i)^2 + 13.67(h/r_i)^3$

   $FNCO = 2.58$

3. Speed of Sound

   $$C_\phi = \sqrt{\frac{\gamma(P+14.7)g}{\rho_f}} = \sqrt{\frac{1.40(39.3+14.7)(32.174)}{3.21E-04(12)}}$$

   $$C_\phi = 794.6 \text{ fps}$$

4. First Radial Acoustic Mode Frequency

   $$f_\lambda = \frac{(FNCO)(C_\phi)}{2\pi r_i} = \frac{12(2.58)(794.6)}{2\pi(4.0)} = 978.84 \text{ Hz}$$

5. First Radial Acoustic Mode Velocity

   $$V_\lambda = \frac{f_\lambda \sigma}{S_{\sigma c}} = \frac{978.84(.400)}{12(0.2)} = 163.14 \text{ fps}$$

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IV. Flow-Induced Stress Calculation for Longitudinal Mode N=1

1. Critical frequency (f_c) at $N=N_c$

$$m_f = 1.0(5.95E-05) + 0.68(5.55E-04)(7/7)$$

$$m_f = 4.37E-04 \text{ slugs}$$

$$m = m_m + m_f = 137.93E-04 + 4.37E-04$$

$$m = 14.23E-03 \text{ slugs}$$

at $N=N_c$, $B_7 = \sqrt{2}$

$$f_c = (f_o)(B_7)$$

$$f_c = \frac{1}{2\pi}\sqrt{\frac{12(13728.54)}{14.23E-03}}\ (\sqrt{2}) = 765.84 \text{ Hz}$$

2. Critical velocity (V_c) at $N=N_c$

$$V_c = \frac{f_c \sigma}{S_{\sigma c}} = \frac{765.84(.400)}{12(0.2)} = 127.64 \text{ fps}$$

3. $V' = V^*/V_c = 20.44/127.64 = 0.160$

4. $SSR = \frac{K_a N_c}{D_m N_p} = \frac{980.61(7)}{9.287(1)} = 739.13 \text{ lbf/in}^2$

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5. $C_{NP} = 1.0$ for $N_p = 1$

*6. $$C^* = \frac{C_1}{C_2 + (V')^2} + \frac{C_3 |\sin(180V')|}{C_4 + (V')^2} + C_5$$

$$C^* = \frac{0.13}{.462 + (.160)^2} + \frac{1.0 |\sin[180(.160)]|}{10 + (.160)^2} + 0.06$$

$$C^* = 0.375$$

*If calculating sin in degrees, use 180; if calculating sin in radians, use π.

7. $$P_D = \frac{\rho_f (V^*)^2}{2g} = \frac{3.21E{-}04(20.44)^2(12)}{2(32.174)} = 0.025 \text{ psi}$$

8. $$DD = \frac{C^* t P_D}{V' SSR\ \delta} = \frac{0.375(.037)(.025)}{0.160(739.13)(.326)} = 9.00E{-}06$$

9. $$EE = 1.0 + 0.1\left(\frac{400}{SSR}\right)^2 = 1.0 + 0.1\left(\frac{400}{739.13}\right)^2 = 1.029$$

10. $C_E = 1.0$ for no elbow present upstream

11. $$FIS = \frac{(EE)\ (DD)\ (E)\ (C_{NP})\ (C_E)}{N_p}$$

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$$FIS = \frac{(1.029)(9.00E-06)(28.5E+06)(1.0)(1.0)}{1.0}$$

FIS = 263.94 psi

12. f(1) = 122.63 Hz < f_2 = 978.84 Hz; therefore

FIS = 263.94 psi for long. mode N=1

NOTE: If f(N) ≥ f_2, then FIS is multiplied by an acoustic factor of five (5).

13. Uncertainty Factor:

The spring rate was estimated using equ. (2) therefore, the uncertainty factor is

UF = 2.0

and the corrected predicted flow-induced stress for longitudinal mode N=1 is:

FISC = (UF)(FIS)

FISC = (2.0)(263.94) = 527.88 psi for long. mode N=1

V. Calculations for Longitudinal Modes N=2 thru 13.

A similar proceedure has been applied to the remaining bellows longitudinal modes (N=2 thru 13) and the results are

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summarized in Table B-1. The convolute bending mode is calculated next as shown below and is also summarized in Table B-1.

VI. Frequency Calculation for Convolute Bending Mode.

$$f_{CB} = \frac{1}{2\pi}\sqrt{\frac{8k}{m_m + .68\ m_{f2}}}$$

where k, m_m, and m_{f2} were previously calculated

$$f_{CB} = \frac{1}{2\pi}\sqrt{\frac{8(13,728.54)(12)}{137.93E{-}04 + .68(5.55E{-}04)}}$$

$$f_{CB} = 1534.89 \text{ Hz}$$

VII. Velocity Range Calculation for Convolute Bending Mode.

$$V_{low} = \frac{f_{CB}\,\sigma}{S_{\sigma u}} = \frac{1534.89(.400)}{12(0.3)} = 170.54 \text{ fps}$$

$$V^* = \frac{f_{CB}\,\sigma}{S_{\sigma c}} = \frac{1534.89(.400)}{12(0.2)} = 255.82 \text{ fps}$$

$$V_{up} = \frac{f_{CB}\,\sigma}{S_{\sigma 1}} = \frac{1534.89(.400)}{12(0.1)} = 511.63 \text{ fps}$$

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VIII. Flow-Induced Stress Calculation for Convolute Bending Mode.

1. $V' = V^*/V_C$

   where V_C was previously calculated

   $V' = 255.82/127.64 = 2.004$

2. $C_{NP} = 1.0$ for $N_p=1$

3. $C^* = 0.4$ for the convolute bending mode

4. $$P_D = \frac{\rho_f (V^*)^2}{2g} = \frac{12(3.21E\text{-}04)(255.82)^2}{2(32.174)}$$

   $P_D = 3.918$ psi

5. $$DD = \frac{C^* t P_D}{V' SSR\,\delta}$$

   where SSR and δ were previously calculated

   $$DD = \frac{(0.4)(.037)(3.918)}{(2.004)(739.13)(.326)} = 1.20E\text{-}04$$

6. EE = 1.029 as previously calculated

7. $C_E = 1.0$ for no elbow present upstream

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8. $$FIS = \frac{(EE)(DD)(E)(C_{NP})(C_E)}{N_P}$$

$$FIS = \frac{(1.029)(1.20E-04)(28.5E+06)(1.0)(1.0)}{1.0}$$

$$FIS = 3,519.18 \text{ psi}$$

9. The convolute bending mode frequency is higher than the first radial acoustic mode frequency

$$f_{CB} = 1534.89 \text{ Hz} > f_2 = 978.84 \text{ Hz}$$

therefore, an acoustic factor of 5 is applied to FIS below.

10. Uncertainty Factors:

The spring rate was estimated using equation (2) and since radial acoustic resonance is predicted, therefore, the uncertainty factor is

$$UF = (2.0)(1.5) = 3.0$$

An acoustic factor of 5 is also applied to FIS to account for an increase in stress levels due to radial acoustic resonance. The corrected

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flow-induced stress for the convolute bending mode is now:

$$FISC = (UF)(5.0)(FIS)$$

$$FISC = (3.0)(5.0)(3{,}519.18)$$

$$FISC = 52{,}788 \text{ psi for the convolute bending mode}$$

IX. Fatigue Assessment

For 21-6-9 steel at -200°F the endurance limit is

$$S_{EL} = 47{,}000 \text{ psi}$$

Conclusion:

From the results summarized in Table B-1 and in Figure B-1 we now compare the predicted FISC values with the S_{EL} value given above. If FISC is less than S_{EL}, infinite life is predicted. If FISC is greater than S_{EL}, the life of the bellows is finite. All 13 longitudinal modes have FISC values less than S_{EL}. The convolute bending mode FISC value is greater than S_{EL} because the first radial acoustic mode resonance occurs at 163.14 fps.

Based on the above, infinite life is predicted until the convolute bending mode frequency is reached. This

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convolute bending mode has a flow excitation range from $V_{low} = 170.54$ fps to $V_{up} = 511.63$ fps with the optimum or most severe flow excitation occuring at $V^* = 255.82$ fps. Therefore, the maximum safe flow velocity should be limited, according to case D in paragraph 2.6, to less than 170.54 fps to maintain a predicted infinite life.

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MODE	CRITICAL VELOCITY (ft/sec)	FREQUENCY (Hz)	ACOUSTIC FACTOR	FIS (psi)	UNCERTAINTY FACTOR	FISC (psi)	LIFE
1	20.44	122.63	1.0	263.94	2.0	527.88	Infinite
2	40.56	243.37	1.0	522	2.0	1045	Infinite
3	60.09	360.53	1.0	719	2.0	1438	Infinite
4	78.79	472.71	1.0	824	2.0	1649	Infinite
5	96.42	578.53	1.0	835	2.0	1671	Infinite
6	112.78	676.70	1.0	768	2.0	1536	Infinite
7	127.64	765.99	1.0	654	2.0	1309	Infinite
8	140.89	845.34	1.0	804	2.0	1609	Infinite
9	152.30	913.78	1.0	937	2.0	1875	Infinite
10	161.75	970.49	1.0	1041	2.0	2082	Infinite
11	169.14	1014.82	5.0	5563	3.0	16,690	Infinite
12	174.37	1046.25	5.0	5784	3.0	17,352	Infinite
13	177.40	1064.43	5.0	5897	3.0	17,690	Infinite
Convolute Bending	255.82	1534.89	5.0	17,596	3.0	52,788	Finite

First Radial Acoustic Mode Resonance: V_λ = 163.14 fps, f_λ = 978.84 Hz

Table B-1. Gaseous Medium Example Summary

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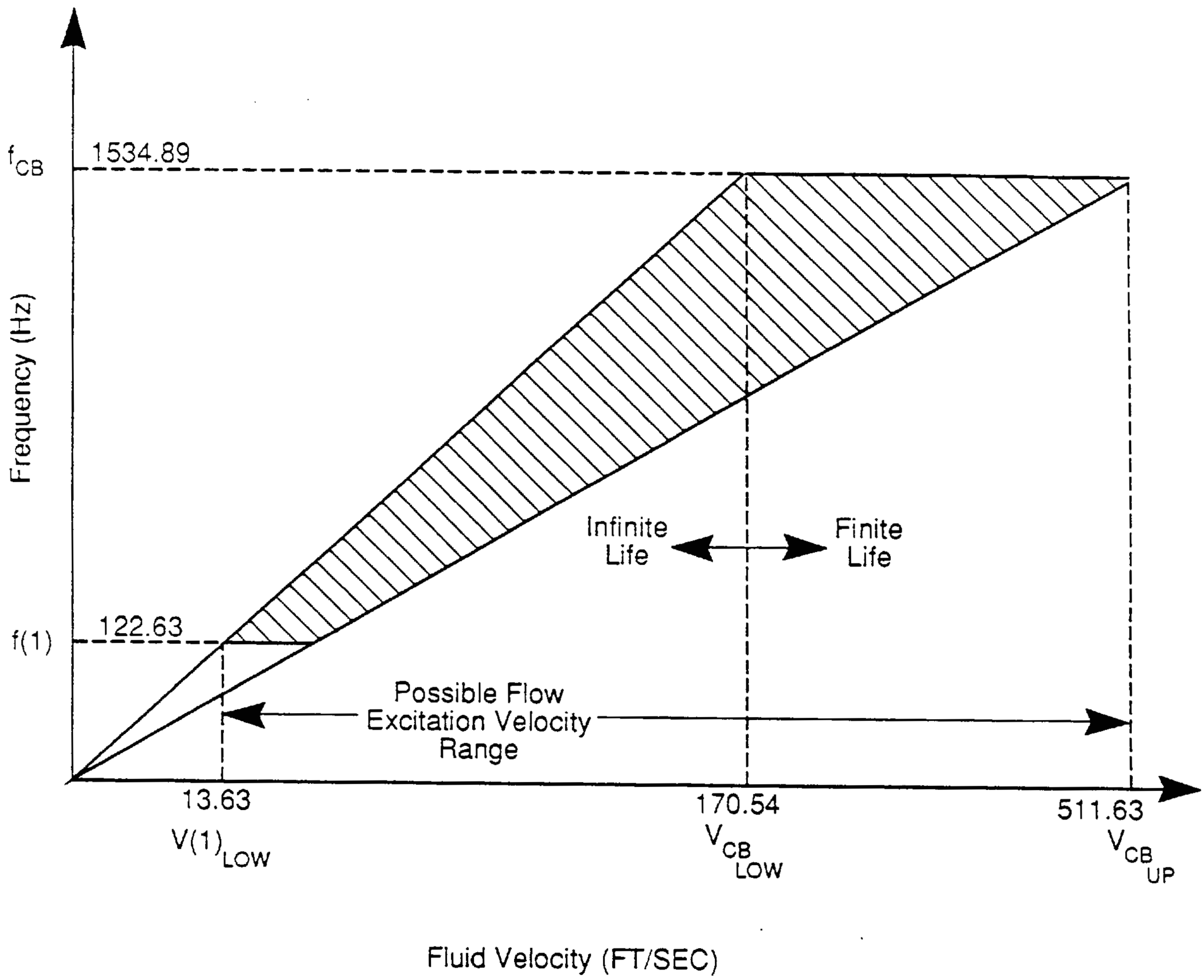

Figure B-1. Frequency vs. Velocity Plot Indicating Flow Excitation Range for Example 1.2

5-8873-0-224

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2.0 FLEXHOSE EXAMPLE PROBLEM

Given: Gaseous helium flowing through a flexible metal hose at 75°F and at 600 psig.

The flexhose is made of 21-6-9 steel and it is assumed the braid will be in contact with all of the convolute crowns. There is no elbow upstream of the flexhose.

FLEXHOSE PARAMETERS

Inside convolute width, σ = 0.072 in.

Inside convolute pitch, λ = 0.104 in.

Mean inside convolute height, h = 0.154 in.

Ply thickness, t = 0.010 in.

Inside diameter, D_i = 1.850 in.

Outside diameter, D_o = 2.198 in.

* Number of convolutes, N_c = 32

Number of plys, N_p = 2

Young's modulus, E = 28.5E+06 psi

Material weight density, ρ_m = 0.282 lbf/cu. in.

* NOTE: This is the actual number of convolutes of the flexhose, however, in the analysis N_c = 1 is used.

Problem: Determine if the flexhose will have infinite life from flow-induced vibration loads when operated at 800 fps flow velocity.

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CALCULATION PROCEDURE:

I. Frequency Calculation for the Three Flexhose Modes.

1. $K_f = D_m E (N_p/N_c)(t/h)^3$

$K_f = 2.024(28.5E+06)(2/1)(.010/.154)^3$

$K_f = 31,588$ lbf/in

$k = 2\ K_f = 2(31,588) = 63,176$ lbf/in

2. $$m_m = \frac{\pi \rho_m t N_p D_m [\pi a + h - 2a]}{g}$$

$a = (\sigma - tN_p)/2 = [.072-.010(2)]/2 = 0.026$ in

$$m_m = \frac{\pi(.282)(.010)(2)(2.024)[\pi(.026)+.154-2(.026)]}{32.174}$$

$m_m = 2.05E-04$ slugs

3. $$m_{IP} = \frac{\pi \rho_f D_m h (2a - tN_p)}{2g}$$

where $\rho_f = \rho_{ref}(P/P_{ref})(T_{ref}/T)(Z_{ref}/Z)$

These terms are explained in example 1.2

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$$\rho_{ref} = \frac{14.7(144)}{385.96(528)(1.0)} = 0.010 \text{ lbf/ft}^3$$

$$\rho_f = \left(\frac{.010}{1728}\right)\left(\frac{600+14.7}{14.7}\right)\left(\frac{528}{535}\right)\left(\frac{1.0}{1.02}\right)$$

$$\rho_f = 2.34E\text{-}04 \text{ lbf/in}^3$$

$$m_{IP} = \frac{\pi(2.34E\text{-}04)(2.024)(.154)[2(.026)-.010(2)]}{2(32.174)}$$

$$m_{IP} = 1.13E\text{-}07 \text{ slugs}$$

4. $$m_{OP} = \frac{0.68\rho_f D_m h^3}{g\,\delta}$$

$$m_{OP} = \frac{0.68(2.34E\text{-}04)(2.024)(.154)^3}{32.174(.032)}$$

$$m_{OP} = 1.14E\text{-}06 \text{ slugs}$$

5. $$f_{IP} = \frac{1}{2\pi}\sqrt{\frac{2k}{m_m + m_{IP}}}$$

$$f_{IP} = \frac{1}{2\pi}\sqrt{\frac{2(63,176)12}{2.05E\text{-}04+1.13E\text{-}07}}$$

$$f_{IP} = 13,684 \text{ Hz} \quad \text{(In-Phase Mode)}$$

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6. $f_{OP} = \frac{1}{2\pi}\sqrt{\frac{2k}{m_m + m_{OP}}}$

$$f_{OP} = \frac{1}{2\pi}\sqrt{\frac{2(63,176)12}{2.05E-04+1.14E-06}}$$

f_{OP} = 13,650 Hz (Out-of-Phase Mode)

7. $f_{CB} = \frac{1}{2\pi}\sqrt{\frac{8k}{m_m + m_{OP}}}$

$$f_{CB} = \frac{1}{2\pi}\sqrt{\frac{8(63,176)12}{2.05E-04+1.14E-06}}$$

f_{CB} = 27,299 Hz (Convolute Bending Mode)

II. Velocity Range Calculation for the Three Flexhose Modes:

1. In-Phase Mode:

$$V_{low} = \frac{(f_{IP})\sigma}{S_{\sigma u}} = \frac{13,684(.072)}{12(0.3)} = 273.7 \text{ fps}$$

$$V^* = \frac{(f_{IP})\sigma}{S_{\sigma c}} = \frac{13,684(.072)}{12(0.2)} = 410.5 \text{ fps}$$

$$V_{up} = \frac{(f_{IP})\sigma}{S_{\sigma 1}} = \frac{13,684(.072)}{12(0.1)} = 821.0 \text{ fps}$$

2. Out-of-Phase Mode:

$$V_{low} = \frac{(f_{OP})\sigma}{S_{\sigma u}} = \frac{13,650(.072)}{12(0.3)} = 273.0 \text{ fps}$$

$$V^{*} = \frac{(f_{OP})\sigma}{S_{\sigma c}} = \frac{13,650(.072)}{12(0.2)} = 409.5 \text{ fps}$$

$$V_{up} = \frac{(f_{OP})\sigma}{S_{\sigma 1}} = \frac{13,650(.072)}{12(0.1)} = 819.0 \text{ fps}$$

3. Convolute Bending Mode:

$$V_{low} = \frac{(f_{CB})\sigma}{S_{\sigma u}} = \frac{27,299(.072)}{12(0.3)} = 546.0 \text{ fps}$$

$$V^{*} = \frac{(f_{CB})\sigma}{S_{\sigma c}} = \frac{27,299(.072)}{12(0.2)} = 819.0 \text{ fps}$$

$$V_{up} = \frac{(f_{CB})\sigma}{S_{\sigma 1}} = \frac{27,299(.072)}{12(0.1)} = 1637.9 \text{ fps}$$

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III. First Radial Acoustic Mode Resonance Calculation

1. $h/r_i = .154/(1.850/2) = .166$

2. $FNCO = 3.8-16.72(h/r_i)^2+13.67(h/r_i)^3$

   $FNCO = 3.40$

3. Speed of Sound:

$$C_\phi = \sqrt{\frac{\gamma(P+14.7)g}{\rho_f}} = \sqrt{\frac{1.66(600+14.7)(32.174)}{2.34E-04(12)}}$$

$$C_\phi = 3419.32 \text{ fps}$$

4. First Radial Acoustic Mode Frequency:

$$f_\lambda = \frac{(FNCO)(C_\phi)}{2\pi r_i} = \frac{12(3.40)(3419.32)}{2\pi(.925)}$$

$$f_\lambda = 24{,}004 \text{ Hz}$$

5. First Radial Acoustic Mode Velocity:

$$V_\lambda = \frac{f_\lambda \sigma}{S_{\sigma c}} = \frac{24{,}004(.072)}{12(0.2)}$$

$$V_\lambda = 720.12 \text{ fps}$$

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IV. Flow-Induced Stress Calculation for the Three Flexhose Modes

1. Critical frequency (f_c)

   The critical frequency corresponds to the out-of-phase mode frequency, f_{OP}, previously calculated.

   $$f_c = f_{OP} = 13,650 \text{ Hz}$$

2. Critical velocity (V_c)

   The critical velocity corresponds to the out-of-phase mode critical velocity, V^*, previously calculated.

   $$V_c = V^* \text{ (out-of-phase)} = 409.5 \text{ fps}$$

3. $V'_{IP} = V^*/V_c = 410.5/409.5 = 1.00$

   $V'_{OP} = V^*/V_c = 409.5/409.5 = 1.00$

   $V'_{CB} = V^*/V_c = 819.0/409.5 = 2.00$

4. $$SSR = \frac{K_f N_c}{D_m N_p} = \frac{31,588(1)}{2.024(2)} = 7803.4 \text{ lbf/in}^2$$

   where N_c=1 for a flexhose

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5. $$C_{NP} = 1.0 - \frac{C_6(\sigma/h)}{1+C_7(V')^2}$$

In-Phase Mode $C_{NP} = .91$

Out-of-Phase Mode $C_{NP} = .91$

Convolute Bending Mode $C_{NP} = .97$

6. C^* for in-phase and out-of-phase modes

$$C^* = \frac{C_1}{C_2+(V')^2} + \frac{C_3|\sin(180V')|}{C_4+(V')^2} + C_5$$

In-Phase Mode $C^* = .15$

Out-of-Phase Mode $C^* = .15$

For the convolute bending mode use $C^* = 0.4$

7. $$P_D = \frac{\rho_f(V^*)^2}{2g}$$

In-Phase Mode $P_D = \frac{(2.34E-04)(410.5)^2 12}{2(32.174)} = 7.35$ psi

Out-of-Phase Mode $P_D = \frac{(2.34E-04)(409.5)^2 12}{2(32.174)} = 7.32$ psi

Convolute Bending Mode $P_D = \frac{(2.34E-04)(819.0)^2 12}{2(32.174)} = 29.27$ psi

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8. $$DD = \frac{C^{*}tP_D}{V' \ SSR \ \delta}$$

In-Phase Mode $$DD = \frac{.15(.010)(7.35)}{1.0(7803.4)(.032)} = 4.41E-05$$

Out-of-Phase Mode DD = 4.40E-05

Convolute Bending Mode DD = 23.44E-05

9. $$EE = 1+0.1\left(\frac{400}{SSR}\right)^2 = 1+0.1\left(\frac{400}{7803.4}\right)^2 = 1.00$$

10. C_E = 1.0 for no elbow present upstream

11. $$FIS = \frac{(EE)(DD)(E)(C_{NP})(C_E)}{N_p}$$

In-Phase Mode FIS = 571.9 psi

Out-of-Phase Mode FIS = 570.6 psi

Convolute Bending Mode FIS = 3240.0 psi

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12. Since the first radial acoustic mode frequency (f_λ) is greater than the in-phase and out-of phase mode frequencies, no acoustic factor of 5 is applied to FIS. However, the convolute bending mode frequency is higher than the first radial acoustic mode frequency.

$$f_{CB} = 27{,}299 \text{ Hz} > f_\lambda = 24{,}004 \text{ Hz}$$

therefore, an acoustic factor of 5 is applied to FIS below.

13. Uncertainty Factor:

For the in-phase and out-of-phase modes the spring rate was estimated using equation (12) and no radial acoustic resonance is predicted, therefore the uncertainty factor is

$$UF = 2.5$$

For the convolute bending mode the spring rate was estimated using equation (12) and since radial acoustic resonance is predicted, therefore the uncertainty factor is

$$UF = (2.5)(1.5) = 3.75$$

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The corrected predicted flow-induced stress for each of the three flexhose modes are:

FISC = (UF)(FIS)

In-Phase Mode FISC = (2.5)(571.9) = 1429.8 psi

Out-of-Phase Mode FISC = (2.5)(570.6) = 1426.5 psi

Convolute Bending Mode FISC = (3.75)(5.0)(3240.0) = 60,750 psi

The results of the three flexhose modes are summarized in Table B-2.

V. Fatigue Assessment

For 21-6-9 steel at 75°F the endurance limit is

S_{EL} = 31,000 psi

Conclusion:

From the results summarized in Table B-2 and in Figure B-2, we now compare the predicted FISC values with the S_{EL} value given above. If FISC is less than S_{EL}, infinite life is predicted. If FISC is greater than S_{EL} then a finite life is predicted. The in-phase and out-of-phase mode FISC values are less than S_{EL}. The convolute

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bending mode FISC value is greater than S_{EL} because radial acoustic resonance occurs at 720.12 fps.

Based on the above infinite life is predicted until the convolute bending mode frequency is reached. This convolute bending mode has a flow excitation range from V_{low}=546.0 fps to V_{up}=1637.9 fps with the optimum or most severe flow excitation occuring at V^*=819.0 fps. Therefore, the maximum safe flow velocity should be limited, according to case D in paragraph 2.6, to less than 546.0 fps to maintain a predicted infinite life. This flexhose will not have infinite life when operated at 800 fps flow velocity.

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MODE	CRITICAL VELOCITY ft/sec	FREQUENCY Hz	ACOUSTIC FACTOR	FIS (psi)	UNCERTAINTY FACTOR	FISC (psi)	LIFE
Out-of-Phase	409.5	13,650	1.0	570.6	2.5	1426.5	Infinite
In-Phase	410.5	13,684	1.0	571.9	2.5	1429.8	Infinite
Convolute Bending	819.0	27,299	5.0	3240.0	3.75	60,750	Finite

First Radial Acoustic Mode Resonance: V_{λ} = 720.12 fps, f_{λ} = 24,004 Hz

Table B-2. Flexhose Example Summary

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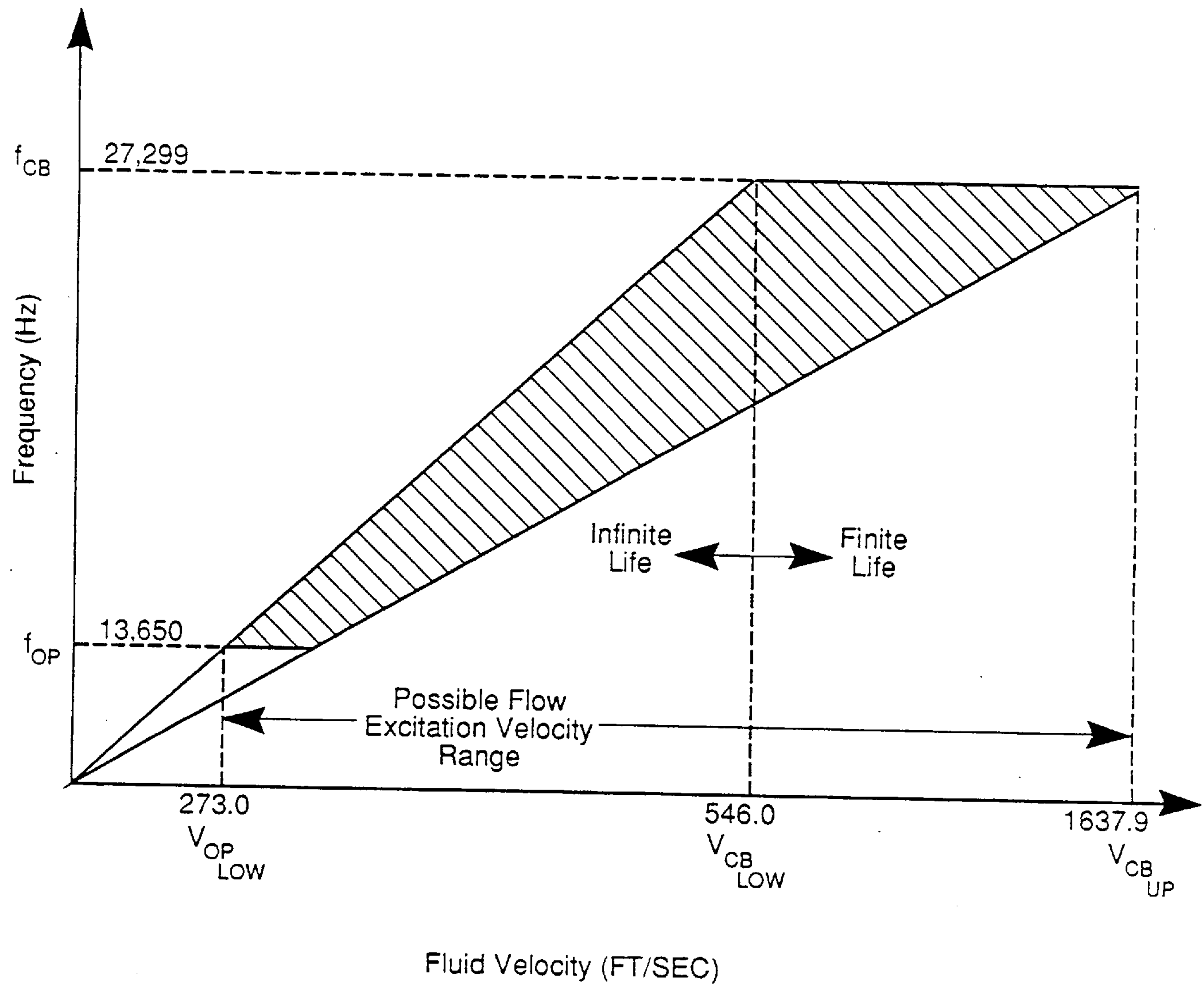

Figure B-2. Frequency vs. Velocity Plot Indicating Flow Excitation Range for Example 2.0

REVISIONS			
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APPENDIX C

BELLOWS FLOW-INDUCED VIBRATION COMPUTER PROGRAM

"BELFIV" Version 3.3

REVISIONS			
SYM	DESCRIPTION	DATE	APPROVAL

1.0 COMPUTER PROGRAM:

The computer program was written to calculate the modal frequencies, flow excitation ranges, and flow-induced stresses for the longitudinal modes and the convolute bending mode in a metal bellows. This program is written to apply to both liquid and gas flows through a metal bellows. In the case of gas flows, it also calculates the first radial acoustic mode frequency and velocity. This program, however, does not conduct flow-induced vibration analysis for a flexhose and does not calculate static stresses in a bellows. The flexhose and static stress analysis have to be done by hand.

This computer program takes into account the uncertainty factors and acoustic factor for flow-induced stress. The output values for flow-induced stress have the appropriate uncertainty factors and acoustic factor already applied.

The computer program gives the user the option of inputing the data from the keyboard or from a data file. An example of an input data file is given in paragraph 1.2. The output data can be saved in a data file or sent to a printer.

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The computer program was written in Fortran language to run interactively with an IBM-XT personal computer. However, this does not limit its use as it can be readily modified to suit the needs of the designer. The differences between the hand calculations and computer calculations are attributed to round-off errors. Differences may also be found when different Fortran compilers are used. The Fortran compiler used for this computer program is the IBM Professional Fortran Compiler, version 1.00 by Ryan-McFarland Corp.

1.1 Comparison of Theoretical and Computer Program Variables:

ANALYSIS	COMPUTER	COMMENT
a	A	Mean convolute radius
B_N	BN	Dimensionless frequency factor
C_E	CE	Elbow factor
C_{NP}	CNP	Damping modifier coefficient
C^*	CST	Force and damping coefficient
C_ϕ	CO	Speed of sound
D_i	DI	Bellows inside diameter
D_o	DO	Bellows outside diameter
D_m	DMEAN	Bellows mean diameter
E	E	Young's modulus of elasticity

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f_{λ}	FREQCO	First radial acoustic mode frequency
f_o	FO	Reference frequency
f_c	FREQC	Critical frequency for mode $N=N_c$
f_{CB}	FREQCB	Convolute bending mode frequency
f(N)	FREQ(MODE)	Modal frequency
g	G	Gravitational acceleration
h	H	Mean inside convolute height
k	K	Elemental spring rate
K_a	KA	Overall bellows spring rate
m	MASS MASSR	Total elemental mass
m_m	MMETAL	Elemental metal mass
m_{f1}	FLUID1	Fluid added mass
m_{f2}	FLUID2	Fluid added mass
m_f	MFLUID MFLUDR	Total elemental fluid added mass
N	MODE	Mode number
$2N_c-1$	NDEG	Number of degrees of freedom for a bellows
N_c	NC	Number of convolutes
N_p	NPLY	Number of plys
P_D	PD	Free stream dynamic pressure
P	P	Fluid pressure

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$S_{\sigma l}$	STLO	Lower Strouhal number
$S_{\sigma u}$	STUP	Upper Strouhal number
$S_{\sigma c}$	STCRIT	Critical Strouhal number
t	T	Ply thickness
V_{low}	V(MODE,1)	Lower limit velocity for mode N
V^*	V(MODE,2)	Critical velocity for mode N
V_{up}	V(MODE,3)	Upper limit velocity for mode N
V_c	VELC	Critical velocity for mode $N=N_c$
V'	VP	Normalized velocity parameter
V_λ	VELCO	First radial acoustic mode velocity
γ	GAMMA	Specific heat ratio for the gas
σ	SIGMA	Inside convolute width
δ	DELTA	Inside convolute gap
λ	LAMBDA	Inside convolute pitch
ρ_f	RHOF	Weight density of fluid
ρ_m	RHOM	Weight density of flexible line material.

1.2 Input Data File Format and Examples

Input File: DFILE

Description of Input File DFILE:

Line 1--TITLE (A70)

This line assigns an identifying label to a particular bellows.

Line 2--JFLAG, NFLUID, NDEG (3I3)

This line enters certain conditions of the bellows.

JFLAG . . . . determines origin of overall spring rate KA

JFLAG=1 . . . . . program will calculate KA

JFLAG=2 . . . . . program will use the given value of KA

NFLUID . . . . flow medium

NFLUID=1 . . . . . gas

NFLUID=2 . . . . . liquid

NDEG . . . . no. of bellows longitudinal degrees of freedom=2NC-1

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Line 3--NC, NPLY, SIGMA, LAMBDA, H, T (6F10.3)

This line enters certain geometric parameters of the bellows.

NC . . . . number of convolutes counted from the outside

NPLY . . . . number of plys

SIGMA . . . . inside convolute width, inches

LAMBDA . . . . inside convolute pitch, inches

H . . . . mean inside convolute height, inches

T . . . . ply thickness, inches

Line 4--DI, DO, E, RHOM, KA, LOVERD (2F10.3, F10.0, 3F10.3)

This line enters certain geometric parameters, matl. properties, and conditions of the bellows.

DI . . . . bellows inside diameter, inches

DO . . . . bellows outside diameter, inches

E . . . . Young's modulus of elasticity, lbs/sq. in.

RHOM . . . . weight density of the material, lbf/cu. in.

KA . . . . bellows overall spring rate, lbf/inch (Input 0.0 if program calculates KA)

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LOVERD . . . . length from termination of elbow to first convolute divided by the I.D. of pipe just before the bellows (input 0.0 if no elbow upstream)

Line 5--This line enters the conditions of the fluid whether a gas or a liquid.

For a liquid--P, TEMP, RHOF (3F10.3)

P . . . . liquid pressure, psig

TEMP . . . . liquid temperature, Faherenheit

RHOF . . . . Weight density of liquid at P and TEMP, lbf/cu. ft.

For a gas--P, TEMP, PREF, TREF, RHOREF (5F10.4)

P . . . . gas pressure, psig

TEMP . . . . gas temperature, Faherenheit

PREF . . . . gas pressure at reference state, psia

TREF . . . . gas temperature at reference state, Fahrenheit

RHOREF . . . . . Weight density of gas at reference state, lbf/cu. ft.

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Line 6--Z, ZREF, GAMMA (3F10.3)

This line enters the gas compressibility factors and specific heat ratio for the gas. This line is only used when the fluid is a gas and is not used when the fluid is a liquid.

Z . . . . gas compressibility factor (non-dim)

ZREF . . . . gas compressibility factor at reference state (non-dim)

GAMMA . . . . specific heat ratio for the gas (non-dim)

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An example of an input file for liquid flow through a metal bellows is shown below. The corresponding output file is shown in paragraph 1.3. This example is identical to the liquid medium example 1.1 presented in Appendix B.

```
LIQUID MEDIUM EXAMPLE 1.1
  1  2 31
    16.000      3.000        0.095    0.148    0.325    0.007
     3.000      3.690    29000000.    0.286    0.000    1.333
    35.000     68.000       62.400
```

An example of an input file for gas flow through a metal bellows is shown below. The corresponding output file is shown in paragraph 1.3. This example is identical to the gaseous medium example 1.2 presented in Appendix B.

```
GASEOUS MEDIUM EXAMPLE 1.2
    1  1 13
       7.000        1.000        0.400     0.726     1.250     0.037
       8.000       10.574    28500000.     0.282     0.000     0.000
     39.3000    -200.0000      14.7000   68.0000    0.0730
       0.982        1.000        1.400
```

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1.3 Output File Examples

On the following pages are the output files corresponding to the two sample input files (liquid and gas).

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LIQUID MEDIUM EXAMPLE 1.1

BELLOWS PARAMETERS

SIGMA(INSIDE CONVOLUTE WIDTH, IN)	0.095
LAMBDA(INSIDE CONVOLUTE PITCH, IN)	0.148
H(MEAN INSIDE CONVOLUTE HEIGHT, IN)	0.325
T(CONVOLUTE THICKNESS PER PLY, IN)	0.007
DI(INSIDE DIAMETER, IN)	3.000
DO(OUTSIDE DIAMETER, IN)	3.690
NC(NUMBER OF CONVOLUTES)	16.000
NPLY(NUMBER OF PLYS)	3.000
E(YOUNG'S MODULUS, LB/SQ.IN)	0.2900E+08
KA(OVERALL SPRING RATE, LBF/IN)	181.735
RHOM(MATERIAL DENSITY, LBF/CU.IN)	0.286

FLUID PARAMETERS

P(PRESSURE, PSIG)	35.000
TEMP(TEMPERATURE, DEG F)	68.000
RHOF(FLUID DENSITY, LBF/CU.IN)	0.3611E-01
NFLUID(1=GAS, 2=LIQUID)	2
CE(ELBOW FACTOR, DIMENSIONLESS)	2.410

THEORETICAL BELLOWS PERFORMANCE

LONG. MODE NO.	FLOW-IND. STRESS WITH U.F., PSI	MODE FREQUENCY HZ	FLOW EXCITATION RANGE, FT/SEC LOWER	CRITICAL	UPPER
1	0.10502E+05	135.638	3.579	5.369	10.738
2	0.21952E+05	256.980	6.781	10.172	20.344
3	0.33304E+05	366.715	9.677	14.516	29.032
4	0.43439E+05	466.738	12.317	18.475	36.950
5	0.51738E+05	558.422	14.736	22.104	44.208
6	0.58014E+05	642.788	16.962	25.444	50.887
7	0.62338E+05	720.613	19.016	28.524	57.049
8	0.64914E+05	792.497	20.913	31.370	62.739
9	0.66006E+05	858.912	22.666	33.999	67.997
10	0.65896E+05	920.234	24.284	36.426	72.852
11	0.64862E+05	976.768	25.776	38.664	77.327
12	0.63155E+05	1028.764	27.148	40.722	81.444
13	0.61000E+05	1076.429	28.406	42.609	85.217
14	0.58585E+05	1119.936	29.554	44.331	88.662
15	0.56062E+05	1159.435	30.596	45.894	91.789
16	0.53554E+05	1195.053	31.536	47.304	94.608
17	0.56752E+05	1226.904	32.377	48.565	97.130
18	0.59664E+05	1255.088	33.120	49.681	99.361
19	0.62256E+05	1279.697	33.770	50.655	101.309
20	0.64508E+05	1300.814	34.327	51.491	102.981
21	0.66410E+05	1318.519	34.794	52.191	104.383
22	0.67960E+05	1332.886	35.173	52.760	105.520
23	0.69160E+05	1343.988	35.466	53.200	106.399
24	0.70015E+05	1351.896	35.675	53.513	107.025
25	0.70532E+05	1356.679	35.801	53.702	107.404
26	0.70719E+05	1358.407	35.847	53.770	107.541
27	0.70583E+05	1357.149	35.814	53.720	107.441
28	0.70132E+05	1352.978	35.704	53.555	107.111
29	0.69374E+05	1345.964	35.518	53.278	106.555
30	0.68316E+05	1336.180	35.260	52.890	105.781
31	0.66969E+05	1323.703	34.931	52.397	104.793
CONVOLUTE BENDING MODE	0.30653E+06	2440.707	64.408	96.611	193.223

CODE IDENT NO	DWG SIZE	20M02540
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GASEOUS MEDIUM EXAMPLE 1.2

BELLOWS PARAMETERS

SIGMA(INSIDE CONVOLUTE WIDTH, IN)	0.400
LAMBDA(INSIDE CONVOLUTE PITCH, IN)	0.726
H(MEAN INSIDE CONVOLUTE HEIGHT, IN)	1.250
T(CONVOLUTE THICKNESS PER PLY, IN)	0.037
DI(INSIDE DIAMETER, IN)	8.000
DO(OUTSIDE DIAMETER, IN)	10.574
NC(NUMBER OF CONVOLUTES)	7.000
NPLY(NUMBER OF PLYS)	1.000
E(YOUNG'S MODULUS, LB/SQ.IN)	0.2850E+08
KA(OVERALL SPRING RATE, LBF/IN)	980.613
RHOM(MATERIAL DENSITY, LBF/CU.IN)	0.282

FLUID PARAMETERS

P(PRESSURE, PSIG)	39.300
TEMP(TEMPERATURE, DEG F)	-200.000
RHOF(FLUID DENSITY, LBF/CU.IN)	0.3209E-03
NFLUID(1=GAS, 2=LIQUID)	1
CE(ELBOW FACTOR, DIMENSIONLESS)	1.000

THEORETICAL BELLOWS PERFORMANCE

LONG. MODE NO.	FLOW-IND. STRESS WITH U.F., PSI	MODE FREQUENCY HZ	FLOW EXCITATION RANGE, FT/SEC LOWER	CRITICAL	UPPER
1	0.52738E+03	122.691	13.632	20.449	40.897
2	0.10446E+04	243.368	27.041	40.561	81.123
3	0.14377E+04	360.526	40.058	60.088	120.175
4	0.16493E+04	472.710	52.523	78.785	157.570
5	0.16705E+04	578.533	64.281	96.422	192.844
6	0.15359E+04	676.693	75.188	112.782	225.564
7	0.13086E+04	765.990	85.110	127.665	255.330
8	0.16087E+04	845.336	93.926	140.889	281.779
9	0.18747E+04	913.776	101.531	152.296	304.592
10	0.20817E+04	970.493	107.833	161.749	323.498
11	0.16690E+05	1014.819	112.758	169.137	338.273
12	0.17352E+05	1046.246	116.250	174.374	348.749
13	0.17690E+05	1064.426	118.270	177.404	354.809
CONVOLUTE BENDING MODE	0.52836E+05	1535.182	170.576	255.864	511.727

FIRST RADIAL ACOUSTIC MODE FREQUENCY= 980.654 HZ

FIRST RADIAL ACOUSTIC MODE VELOCITY= 163.442 FT/SEC

CODE IDENT NO	DWG SIZE	20M02540
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1.4 Program Listing for BELFIV

```
C
C....... THIS IS THE BELFIV PROGRAM ------- VERSION 3.3
C
C        THIS PROGRAM CALCULATES THE MODAL FREQUENCIES, FLOW EXCITATION
C        RANGES, AND THE FLOW-INDUCED STRESSES FOR THE LONGITUDINAL
C        MODES AND THE CONVOLUTE BENDING MODE IN A METAL BELLOWS.
C        THIS PROGRAM APPLIES TO BOTH LIQUID AND GAS FLOWS.  IN THE
C        CASE OF GAS FLOWS, IT ALSO CALCULATES THE FIRST RADIAL ACOUSTIC
C        MODE FREQUENCY AND VELOCITY.  THIS PROGRAM, HOWEVER, DOES NOT
C        CONDUCT FLOW-INDUCED VIBRATION ANALYSIS FOR A FLEXHOSE AND
C        DOES NOT CALCULATE STATIC STRESSES IN A BELLOWS.
C
C        THIS PROGRAM WAS WRITTEN TO OPERATE ON AN IBM-XT COMPUTER
C        WITH A FORTRAN COMPILER WRITTEN BY RYAN-McFARLAND CORPORATION.
C
C        THE FOLLOWING PARAMETERS ARE USED:
C
C        JFLAG = 1(COMPUTE KA), 2(USE GIVEN KA).  KA IS THE OVERALL
C           BELLOWS SPRING RATE, LBF/IN
C        NFLUID = 1(GAS), 2(LIQUID)
C        NDEG = NUMBER OF BELLOWS LONGITUDINAL DEGREES OF FREEDOM, 2*NC-1
C        NC = NUMBER OF BELLOWS CONVOLUTES COUNTED FROM THE OUTSIDE
C        SIGMA = INSIDE CONVOLUTE WIDTH, IN.
C        LAMBDA = INSIDE CONVOLUTE PITCH, IN.
C        H = MEAN INSIDE CONVOLUTE HEIGHT, IN.
C        T = CONVOLUTE THICKNESS PER PLY, IN.
C        NPLY = NUMBER OF PLYS IN THE BELLOWS CONVOLUTES
C        DI = BELLOWS INSIDE DIAMETER, IN.
C        DO = BELLOWS OUTSIDE DIAMETER, IN.
C        E = YOUNG'S MODULUS OF THE BELLOWS MATERIAL, LB/SQ IN.
C        RHOM = WEIGHT DENSITY OF THE BELLOWS MATERIAL, LBF/CU IN.
C        LOVERD = LENGTH FROM TERMINATION OF ELBOW TO FIRST CONVOLUTE
C           DIVIDED BY THE I.D. OF PIPE JUST BEFORE THE BELLOW, NON-DIM.
C        CE = DIMENSIONLESS ELBOW FACTOR
C        IF NFLUID = 1(GAS), THE PERFECT GAS EQUATION OF STATE IS USED
C           FOR CALCULATING GAS DENSITY AT THE STATE DEFINED BY P AND TEMP.
C           IT IS ASSUMED THAT THE GAS PROPERTIES ARE KNOWN AT A REFERENCE
C           STATE DEFINED BY RHOREF, PREF, AND TREF.
C           P = GAS PRESSURE, PSIG
C           TEMP = GAS TEMPERATURE, DEG. F.
C           PREF AND TREF = REFERENCE GAS STATE, PSIA AND DEG. F.
C           RHOREF = WEIGHT DENSITY OF GAS AT REFERENCE STATE, LBF/CU FT.
C           Z = GAS COMPRESSIBILITY FACTOR, NON-DIM.
C           ZREF = GAS COMPRESSIBILITY FACTOR AT REFERENCE STATE, NON-DIM.
C           GAMMA = SPECIFIC HEAT RATIO FOR THE GAS, NON-DIM.
C        IF NFLUID = 2(LIQUID), THE LIQUID DENSITY MUST BE KNOWN APRIORI
C           AT THE LIQUID STATE (P AND TEMP).
C           P = LIQUID PRESSURE, PSIG
C           TEMP = LIQUID TEMPERATURE, DEG. F.
C           RHOF = WEIGHT DENSITY OF LIQUID AT P AND TEMP, LBF/CU FT.
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C
      IMPLICIT REAL(A-H,O-Z)
      REAL MODER,MASS,MFLUID,MFLUDR,MMETAL,MASSR
      REAL KA,K,N1,LOVERD,NC,NPLY,LAMBDA,FLUID1,FLUID2
      INTEGER*2 ANS,DEV
      CHARACTER*5 TITLE(80),DFILE(20),OFILE(20)
      DIMENSION FREQ(75),V(75,3),FISC(75)
```

```
C
C...... SET  DATA  FILES
C
      WRITE(6,5)
    5 FORMAT(1X,'DID YOU SET OUTPUT AND INPUT FILE NAMES ?'/
     $' (YES=1,NO=2)'/)
      READ(5,*) NAM
      GO TO (20,10),NAM
   10 WRITE(6,15)
   15 FORMAT(1X,'RETURN TO DOS ENVIRONMENT TO SET FILE NAMES.'/
     $' (SET DFILE=input.DAT)'/' (SET OFILE=output.DAT)')
      GO TO 2000
   20 WRITE(6,25)
   25 FORMAT(1X,'WILL THE INPUT DATA BE FROM THE KEYBOARD OR FILE ?'/
     $' KEYBOARD=1,FILE=2'/)
      READ(5,*) INP
      GO TO (30,220),INP
C
C...... INPUT  DATA  FROM  KEYBOARD
C
   30 WRITE(6,35)
   35 FORMAT(1X,'HOW DO YOU IDENTIFY THIS BELLOWS ?',/)
      READ(5,40)(TITLE(I),I=1,70)
   40 FORMAT(70A1)
      WRITE(6,45)
   45 FORMAT(1X,'COMPUTE OR USE GIVEN SPRING RATE ?'/
     $' 1 (COMPUTE KA), 2 (USE GIVEN KA)'/)
      READ(5,*) JFLAG
      GO TO (60,50),JFLAG
   50 WRITE(6,55)
   55 FORMAT(1X,'OVERALL BELLOWS SPRING RATE, LBF/IN. ?'/)
      READ(5,*) KA
   60 CONTINUE
      WRITE(6,65)
   65 FORMAT(1X,'IS THE FLUID A GAS OR LIQUID ?'/
     $' 1 (GAS), 2 (LIQUID)'/)
      READ(5,*) NFLUID
      WRITE(6,70)
   70 FORMAT(1X,'NUMBER OF BELLOWS CONVOLUTES COUNTED FROM THE OUTSIDE
     $?'/)
      READ(5,*) NC
      NDEG = 2*NC-1
      WRITE(6,75)
   75 FORMAT(1X,'NUMBER OF PLYS IN THE BELLOWS CONVOLUTES ?'/)
      READ(5,*) NPLY
      WRITE(6,80)
   80 FORMAT(1X,'INSIDE CONVOLUTE WIDTH, IN. ?'/)
      READ(5,*) SIGMA
      WRITE(6,85)
   85 FORMAT(1X,'INSIDE CONVOLUTE PITCH, IN. ?'/)
      READ(5,*) LAMBDA
      WRITE(6,90)
   90 FORMAT(1X,'MEAN INSIDE CONVOLUTE HEIGHT, IN. ?'/)
      READ(5,*) H
      WRITE(6,95)
   95 FORMAT(1X,'CONVOLUTE THICKNESS PER PLY, IN. ?'/)
      READ(5,*) T
      WRITE(6,100)
  100 FORMAT(1X,'BELLOWS INSIDE DIAMETER, IN. ?'/)
      READ(5,*) DI
      WRITE(6,105)
  105 FORMAT(1X,'BELLOWS OUTSIDE DIAMETER, IN. ?'/)
```

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```
      READ(5,*) DO
      WRITE(6,110)
  110 FORMAT(1X,'YOUNGS MODULUS FOR BELLOWS MATERIAL, LB/SQ IN. ?'/)
      READ(5,*) E
      WRITE(6,115)
  115 FORMAT(1X,'WEIGHT DENSITY OF THE BELLOWS MATERIAL, LBF/CU IN. ?'/)
      READ(5,*) RHOM
      WRITE(6,120)
  120 FORMAT(1X,'LENGTH FROM TERMINATION OF ELBOW TO FIRST CONVOLUTE'/
     $' DIVIDED BY THE I.D. OF PIPE JUST BEFORE THE BELLOW ?(INPUT 0 IF
     $NO ELBOW)'/)
      READ(5,*) LOVERD
      GO TO (125,160),NFLUID
  125 WRITE(6,130)
  130 FORMAT(1X,'GAS PRESSURE, PSIG ?'/)
      READ(5,*) P
      WRITE(6,135)
  135 FORMAT(1X,'GAS TEMPERATURE, DEG. F ?'/)
      READ(5,*) TEMP
      WRITE(6,140)
  140 FORMAT(1X,'GAS PRESSURE AT REFERENCE STATE, PSIA ?'/)
      READ(5,*) PREF
      WRITE(6,145)
  145 FORMAT(1X,'GAS TEMPERATURE AT REFERENCE STATE, DEG. F ?'/)
      READ(5,*) TREF
      WRITE(6,150)
  150 FORMAT(1X,'WEIGHT DENSITY OF GAS AT REFERENCE STATE, LBF/CU FT. ?'
     $/)
      READ(5,*) RHOREF
      WRITE(6,151)
  151 FORMAT(1X,'GAS COMPRESSIBILITY FACTOR, NON-DIM. ?'/)
      READ(5,*) Z
      WRITE(6,152)
  152 FORMAT(1X,'GAS COMPRESSIBILITY FACTOR AT REFERENCE STATE, NON-DIM.
     $ ?'/)
      READ(5,*) ZREF
      WRITE(6,155)
  155 FORMAT(1X,'SPECIFIC HEAT RATIO FOR THE GAS, NON-DIM. ?'/)
      READ(5,*) GAMMA
      GO TO 180
  160 WRITE(6,165)
  165 FORMAT(1X,'LIQUID PRESSURE, PSIG ?'/)
      READ(5,*) P
      WRITE(6,170)
  170 FORMAT(1X,'LIQUID TEMPERATURE, DEG.F ?'/)
      READ(5,*) TEMP
      WRITE(6,175)
  175 FORMAT(1X,'WEIGHT DENSITY OF LIQUID AT THE LIQUID STATE (P AND TEM
     $P), LBF/CU FT. ?'/)
      READ(5,*) RHOF
C
C...... SAVE  INPUT  DATA  FROM  KEYBOARD
C
  180 WRITE(6,185)
  185 FORMAT(1X,'DO YOU WISH TO SAVE INPUT DATA ? (YES=1, NO=2)'/)
      READ(5,*) NSAVE
      IF(NSAVE .EQ. 2) GO TO 250
      OPEN (UNIT=10, FILE='DFILE')
      WRITE(10,225)(TITLE(I),I=1,70)
      WRITE(10,230)JFLAG,NFLUID,NDEG
```

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```
      WRITE(10,235)NC,NPLY,SIGMA,LAMBDA,H,T
      IF (JFLAG .EQ. 1) KA=0.0
      WRITE(10,236)DI,DO,E,RHOM,KA,LOVERD
      GO TO (200,210),NFLUID
  200 WRITE(10,241)P,TEMP,PREF,TREF,RHOREF
      WRITE(10,242)Z,ZREF,GAMMA
      GO TO 250
  210 WRITE(10,242)P,TEMP,RHOF
      GO TO 250
C
C...... INPUT  DATA  FROM  FILE
C
  220 OPEN (UNIT=7, FILE='DFILE')
      READ(7,225)(TITLE(I),I=1,70)
  225 FORMAT(70A1)
      READ(7,230)JFLAG,NFLUID,NDEG
  230 FORMAT(3I3)
      READ(7,235)NC,NPLY,SIGMA,LAMBDA,H,T
      READ(7,236)DI,DO,E,RHOM,KA,LOVERD
  235 FORMAT(6F10.3)
  236 FORMAT(2F10.3,F10.0,3F10.3)
      GO TO (240,245),NFLUID
  240 READ(7,241)P,TEMP,PREF,TREF,RHOREF
  241 FORMAT(5F10.4)
      READ(7,242)Z,ZREF,GAMMA
  242 FORMAT(3F10.3)
      GO TO 250
  245 READ(7,242)P,TEMP,RHOF
  250 CONTINUE
      PI=3.1415927
      G=32.174049
      DMEAN=(DI+DO)/2.0
      GO TO (400,405),JFLAG
C
C...... CALCULATION OF SPRING RATE
C
  400 KA=DMEAN*E*(NPLY/NC)*(T/H)**3
  405 K=2.*NC*KA
C
C...... CALCULATION OF METAL MASS AND FLUID MASS
C
      A=(SIGMA-T*NPLY)/2.
      MMETAL=PI*RHOM*T*NPLY*DMEAN*(PI*A+H-2.*A)/G
      GO TO (410,415),NFLUID
  410 RHOF=(RHOREF/1728.)*((P+14.7)/PREF)*((TREF+460.)/(TEMP+460.))*
     $(ZREF/Z)
      GO TO 420
  415 RHOF=RHOF/1728.
  420 FLUID1=PI*RHOF*DMEAN*H*(2.*A-T*NPLY)/(2.*G)
      DELTA=LAMBDA-SIGMA
      FLUID2=RHOF*DMEAN*(H**3)/(G*DELTA)
C
C...... CALCULATION OF CRITICAL FREQUENCY AND VELOCITY (AT MODE N=NC)
C
      STUP=0.3
      STLO=0.1
      STCRIT=0.2
      MODER=NC
      MFLUDR=1.0*FLUID1 + 0.68*(FLUID2*MODER)/NC
      MASSR=MFLUDR+MMETAL
```

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```
      FO=(1./(2.*PI))*SQRT(12.*K/MASSR)
      FREQC=FO*SQRT(2.)
      VELC=FREQC*SIGMA/(STCRIT*12.)
C
C...... CALCULATION OF FREQ. AND VEL. RANGE FOR LOGITUDINAL MODES
C
      DO 440 MODE=1,NDEG
      MFLUID=1.0*FLUID1 + 0.68*FLUID2*(MODE/NC)
      MASS=MFLUID+MMETAL
      BN=SQRT(2.*(1.+COS((PI*(2.*NC-MODE))/(2.*NC))))
      FO=(1./(2.*PI))*SQRT(12.*K/MASS)
      FREQ(MODE)=FO*BN
      DO 440 J=1,3
      GO TO (425,430,435),J
  425 V(MODE,J)=FREQ(MODE)*SIGMA/(STUP*12.)
      GO TO 440
  430 V(MODE,J)=FREQ(MODE)*SIGMA/(STCRIT*12.)
      GO TO 440
  435 V(MODE,J)=FREQ(MODE)*SIGMA/(STLO*12.)
  440 CONTINUE
C
C...... CALCULATION OF FIRST RADIAL ACOUSTIC MODE (GAS MEDIA ONLY)
C
      GO TO (600,615), NFLUID
  600 RI=DI/2.
      HRI=H/RI
      CO=SQRT(GAMMA*(P+14.7)*G/(RHOF*12.))
      IF(HRI.LE.0.40) GO TO 605
      FNCO=-.336+.935*(RI/H)
      GO TO 610
  605 FNCO=3.8-16.72*(HRI**2)+13.67*(HRI**3)
  610 FREQCO=12.*FNCO*CO/(2.*PI*RI)
      QADJUS=5.0
      VELCO=FREQCO*SIGMA/(STCRIT*12.)
C
C...... CALCULATION OF FLOW-INDUCED STRESS FOR LONGITUDINAL MODES
C
  615 SSR=KA*NC/(DMEAN*NPLY)
      C1=.13
      C2=.462
      C3=1.0
      C4=10.0
      C5=.06
      C6=1.25
      C7=5.5
      N1=1.0
      IF(LOVERD.EQ.0.0) N1=0.0
      CE=1.+(N1*4.7/(2.+LOVERD))
      DO 655 MODE=1,NDEG
      VP=V(MODE,2)/VELC
      IF(NPLY.GT.1.) GO TO 620
      CNP=1.0
      GO TO 625
  620 CNP=1.0-((C6*SIGMA/H)/(1.0+C7*VP**2))
  625 BB=C1/(C2+VP**2)
      CC=C3*ABS(SIN(PI*VP))/(C4+VP**2)
      CST=BB+CC+C5
      PD=12.0*RHOF*(V(MODE,2)**2)/(2.0*G)
      DD=CST*T*PD/(VP*SSR*DELTA)
      EE=1.0+0.1*((400.0/SSR)**2)
```

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```
      FIS=EE*DD*E*CNP*CE/NPLY
C
C...... UNCERTAINTY FACTORS FOR STRESS (LONGITUDINAL MODES)
C
      GO TO (630,635), NFLUID
  630 IF (FREQ(MODE).GE.FREQCO) FIS=FIS*QADJUS*1.5
  635 CONTINUE
      GO TO (640,645), JFLAG
  640 FIS=FIS*2.0
      GO TO 650
  645 FIS=FIS*1.5
  650 CONTINUE
      FISC(MODE)=FIS
  655 CONTINUE
C
C...... CALCULATION OF FREQ. AND VEL. RANGE FOR CONVOLUTE BENDING MODE
C
      FREQCB=(1./(2.*PI))*SQRT(8.*K*12./(MMETAL+.68*FLUID2))
      VCBLOW=FREQCB*SIGMA/(STUP*12.)
      VCBSTAR=FREQCB*SIGMA/(STCRIT*12.)
      VCBUP=FREQCB*SIGMA/(STLO*12.)
C
C...... CALCULATION OF FLOW-INDUCED STRESS FOR CONVOLUTE BENDING MODE
C
      VP=VCBSTAR/VELC
      IF (NPLY.GT.1) GO TO 660
      CNP=1.0
      GO TO 665
  660 CNP=1.0-((C6*SIGMA/H)/(1.0+C7*VP**2))
  665 CST=0.4
      PD=12.0*RHOF*(VCBSTAR**2)/(2.0*G)
      DD= CST*T*PD/(VP*SSR*DELTA)
      FIS=EE*DD*E*CNP*CE/NPLY
C
C...... UNCERTAINTY FACTORS FOR STRESS (CONVOLUTE BENDING MODE)
C
      GO TO (670,675), NFLUID
  670 IF (FREQCB.GE.FREQCO) FIS=FIS*QADJUS*1.5
  675 CONTINUE
      GO TO (680,685), JFLAG
  680 FIS=FIS*2.0
      GO TO 690
  685 FIS=FIS*1.5
  690 CONTINUE
      FISCB=FIS
C
C...... OUTPUT DATA
C
      DEV=6
  800 WRITE(DEV,805)(TITLE(I),I=1,70)
  805 FORMAT(1X,70A1)
      WRITE(DEV,840) SIGMA,LAMBDA,H,T,DI,DO,NC,NPLY,E
      WRITE(DEV,845) KA,RHOM,P,TEMP,RHOF,NFLUID,CE
      WRITE(DEV,850)
      DO 810 MODE=1,NDEG
  810 WRITE(DEV,855)MODE,FISC(MODE),FREQ(MODE),V(MODE,1),V(MODE,2),
     $V(MODE,3)
      WRITE(DEV,856)FISCB,FREQCB,VCBLOW,VCBSTAR,VCBUP
      GO TO (815,820), NFLUID
  815 WRITE(DEV,860) FREQCO
```

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```
      WRITE(DEV,865) VELCO
  820 CONTINUE
  825 IF (DEV.EQ.8 .OR. DEV.EQ.9) GO TO 835
      WRITE(6,830)
  830 FORMAT(//,1X,'WHERE DO YOU WANT OUTPUT SENT? 0-EXIT,1-FILE,2-PRINT
     $ER'/)
      READ(5,*)ANS
      IF(ANS.EQ.0) GO TO 2000
      IF(ANS.EQ.1) DEV=8
      OPEN (UNIT=8, FILE='OFILE')
      IF(ANS.EQ.2) DEV=9
      OPEN (UNIT=9, FILE='LPT1')
      GO TO 800
  835 CONTINUE
  840 FORMAT(/,29X,18HBELLOWS PARAMETERS,//
     $       19X,33HSIGMA(INSIDE CONVOLUTE WIDTH, IN),4X,F6.3,/
     $       19X,34HLAMBDA(INSIDE CONVOLUTE PITCH, IN),3X,F6.3,/
     $       19X,35HH(MEAN INSIDE CONVOLUTE HEIGHT, IN),2X,F6.3,/
     $       19X,34HT(CONVOLUTE THICKNESS PER PLY, IN),3X,F6.3,/
     $       19X,23HDI(INSIDE DIAMETER, IN),14X,F6.3,/
     $       19X,24HDO(OUTSIDE DIAMETER, IN),13X,F6.3,/
     $       19X,24HNC(NUMBER OF CONVOLUTES),12X,F7.3,/
     $       19X,20HNPLY(NUMBER OF PLYS),16X,F7.3,/
     $       19X,28HE(YOUNG'S MODULUS, LB/SQ.IN),4X,E11.4)
  845 FORMAT(19X,31HKA(OVERALL SPRING RATE, LBF/IN),1X,F11.3,/
     $       19X,33HRHOM(MATERIAL DENSITY, LBF/CU.IN),3X,F7.3,//
     $       30X,16HFLUID PARAMETERS,//
     $       19X,17HP(PRESSURE, PSIG),19X,F7.3,/
     $       19X,24HTEMP(TEMPERATURE, DEG F),11X,F8.3,/
     $       19X,30HRHOF(FLUID DENSITY, LBF/CU.IN),2X,E11.4,/
     $       19X,23HNFLUID(1=GAS, 2=LIQUID),19X,I1,/
     $       19X,'CE(ELBOW FACTOR, DIMENSIONLESS)',6X,F6.3///
     $       25X,'THEORETICAL BELLOWS PERFORMANCE',/)
  850 FORMAT(2X,78HLONG.    FLOW-IND. STRESS    MODE FREQUENCY      FLOW
     $ EXCITATION RANGE, FT/SEC,/,1X,8HMODE NO.,3X,14HWITH U.F., PSI,
     $11X,2HHZ,13X,5HLOWER,5X,8HCRITICAL,4X,5HUPPER,/)
  855 FORMAT(3X,I2,8X,E11.5,7X,F11.3,5X,3F11.3)
  856 FORMAT(//,1X,9HCONVOLUTE,/,2X,7HBENDING,/,3X,4HMODE,6X,E11.5,7X,
     $F11.3,5X,3F11.3)
  860 FORMAT(//,3X,'FIRST RADIAL ACOUSTIC MODE FREQUENCY=',F9.3,1X,
     $'HZ'/)
  865 FORMAT(3X,'FIRST RADIAL ACOUSTIC MODE VELOCITY=',F9.3,1X,
     $'FT/SEC'/)
 2000 CONTINUE
      END
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